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UNIVERZITET CRNE GORE

SENATU

CENTAR ZA DOKTORSKE STUDIJE

U prilogu akta dostavljam Odluke sa LXXV sjednice Vijeća Prirodno-matematičkog fakuteta održane 24.12.2021. godine.

Dekan,
Prof. dr Predrag Miranović

ISPUNJENOST USLOVA DOKTORANDA

OPŠTI PODACI O DOKTORANDU			
Titula, ime, ime roditelja, prezime	MSc, Edin (Hasib) Liđan		
Fakultet	Prirodno-matematički fakultet, Podgorica		
Studijski program	Matematika		
Broj indeksa	1/15		
NAZIV DOKTORSKE DISERTACIJE			
Na službenom jeziku	Topološke karakteristike popločavanja generalisanih poliominima		
Na engleskom jeziku	Topological caracteristics of generalized polyomino tilings		
Naučna oblast	Kombinatorika, Topološka kombinatorika		
MENTOR/MENTORI			
Prvi mentor	Viši naučni saradnik, Dr Đorđe Baralić	MI SANU, Srbija	Algebarska topologija, Kombinatorika, Projektivna i računarska geometrija,Teorija mnogostrukosti, Diskretna matematika
Drugi mentor	(Titula, ime i prezime)	(Ustanova i država)	(Naučna oblast)
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Dr Đorđe Baralić, viši naučni saradnik - mentor		MI SANU, Srbija	Algebarska topologija, Kombinatorika
Dr Žana Kovijanić Vukićević, red. prof.		PMF Podgorica, Crna Gora	Diskretna matematika, Kombinatorika
Dr Vladimir Božović, red. prof.		PMF Podgorica, Crna Gora	Algebra, Diskretna matematika
Dr Rade Živaljević, naučni savjetnik		MI SANU, Srbija	Algebarska topologija, Kombinatorika
Datum značajni za ocjenu doktorske disertacije			
Sjednica Senata na kojoj je data saglasnost na ocjenu teme i kandidata		12. 02. 2019. g.	

Dostavljanja doktorske disertacije organizacionoj jedinici i saglasanost mentora	17. 12. 2021. g.
Sjednica Vijeća organizacione jedinice na kojoj je dat prijedlog za imenovanje komisija za pregled i ocjenu doktorske disertacije	24. 12. 2021. g.

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E. Liđan, Đ Baralić, Homology of polyomino tilings on flat surfaces, Applicable Analysis and Discrete Mathematics, Belgrade, 2021.
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https://arxiv.org/abs/2103.04404
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- ☐ Znanstveni seminar: Seminar za kombinatoriku i diskretnu matematiku, Prirodno-matematički fakultet, Zagreb (Hrvatska)
- ☐ Heidelberg laureate forum, Heidelberg (Njemačka)
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Datum i ovjera (pečat i potpis odgovorne osobe)

U Podgorici
24. 12. 2021. g.

MP

DEKAN

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2. Odluku o imenovanju komisije za pregled i ocjenu doktorske disertacije
3. Kopiju rada publikovanog u časopisu sa odgovarajuće liste
4. Biografiju i bibliografiju kandidata
5. Biografiju i bibliografiju članova komisije za pregled i ocjenu doktorske disertacije sa potvrdom o izboru u odgovarajuće akademsko zvanje i potvrdom da barem jedan član komisije nije u radnom odnosu na Univerzitetu Crne Gore

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Datum: 21.12.2021.god

Na osnovu člana 33 Zakona o upravnom postupku, nakon uvida u službenu evidenciju, Prirodno-matematički fakultet izdaje

POTVRDU

MSc Edin Liđan, student doktorskih studija na Prirodno-matematičkom fakultetu u Podgorici, dana 17.12.2021.godine dostavio je ovom fakultetu doktorsku disertaciju pod nazivpm "***Topološke karakteristike popločavanja generalisanih poliominima",*** nadalje postupanje.

DEKAN
Prof. dr Predrag Miranović

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Na osnovu člana 64 Statuta Univerziteta Crne Gore, a u vezi sa članom 41 stav 1 Pravila doktorskih studija, na LXXV sjednici Vijeća PMF-a od 24.12.2021.godine donijeta je

ODLUKA

I

Utvrđuje se da su ispunjeni uslovi iz člana 38 Pravila doktorskih studija za doktoranda Edina Liđana

II

Predlaže se Odboru za doktorske studije sastav komisije za ocjenu doktorske disertacije:

1. Prof. dr Svjetlana Terzić, redovni profesor na PMF-u, predsjednik, (naučna oblast: Algebarska topologija i Diferencijalna geometrija);
2. Dr Đorđe Baralić, viši naučni savjetnik, mentor, MI SANU (naučna oblast: Algebarska topologija, Kombinatorika);
3. Prof. dr Žana Kovijanić Vukićević, redovni profesor na PMF-u (naučna oblast: Diskretna matematika, Kombinatorika), član;
4. Prof. dr Vladimir Božović, redovni profesor na PMF-u (naučna oblast: Algebra, Diskretna matematika), član
5. Dr Rade Živaljević, naučni saradnik MI SANU; Srbija (naučna oblast: Algebarska topologija, Kombinatorika) član.

III

Odluka se dostavlja Odboru za doktorske studije Univerziteta Crne Gore.

DEKAN
Prof. dr Predrag Miranović

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Homology of polyomino tilings on flat surfaces

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The homology group of a tiling introduced by M. Reid is studied for certain topological tilings. As in the planar case, for finite square grids on topological surfaces, the method of homology groups, namely the non-triviality of some specific element in the group allows a 'coloring proof' of impossibility of a tiling. Several results about the non-existence of polyomino tilings on certain square-tiled surfaces are proved in the paper.

Keywords: polyomino, combinatorial grids, flat surfaces, homology group of tiling

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HOMOLOGY OF POLYOMINO TILINGS ON FLAT SURFACES

Edin Liđan and Đorđe Baralić

The homology group of a tiling introduced by M. Reid is studied for certain topological tilings. As in the planar case, for finite square grids on topological surfaces, the method of homology groups, namely the non-triviality of some specific element in the group allows a 'coloring proof' of impossibility of a tiling. Several results about the non-existence of polyomino tilings on certain square-tiled surfaces are proved in the paper.

1. INTRODUCTION

Recreational mathematics comprises various subjects including combinatorial games, puzzles, card tricks, art, etc. Its problems are typically easily understood by a general audience, yet their solution often requires rigorous research. Indeed, a significant number of mathematical disciplines have been grounded on ideas sparked by challenges from recreational mathematics. For example, graph theory has its roots in the solution of the problem of The Seven Bridges of Königsberg, and magic squares contributed to the foundations of combinatorial designs.

A *polyomino* is a planar geometric figure formed by joining one or more identical squares edge-to-edge. It may also be regarded as a finite subset of the regular square grid with a connected interior. A polyomino consisting of exactly n cells is called an *n-omino*. Polyomino shapes for $n \leq 5$ are illustrated in Figures 1, 2 and 3. Some polyominoes were named after letters of the alphabet closely resembling them, as can be seen in Figures 2 and 3. They were popularized by Salmon Golomb who wrote the first monograph on polyominoes [8], and by Martin

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Gardner in his Scientific American columns "Mathematical Games", see [6]. In fact, the word polyomino was coined by Golomb in [7]. Today they are one of the most popular subjects of recreational mathematics, being of great interest to not only mathematicians but physicists, biologists, and computer scientists as well. For more information, we refer the reader to surveys [2] and [3].

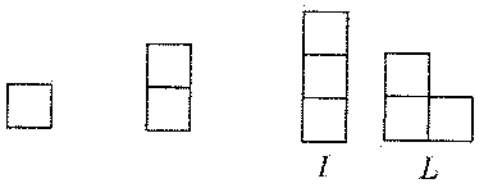

Figure 1: Monomino, domino and trominoes

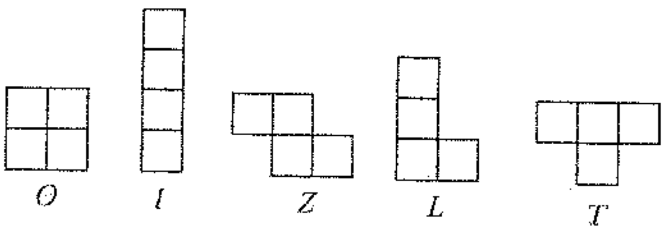

Figure 2: Tetrominoes

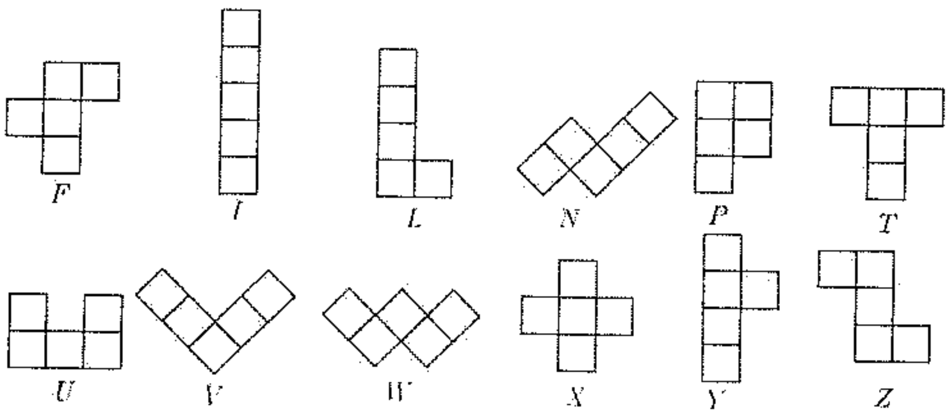

Figure 3: Pentominoes

The polyomino tiling problem asks whether it is possible to properly tessellate a finite region of cells, say M, with polyomino shapes from a given set $\mathcal{T}$. There are numerous generalizations of this question for symmetric and asymmetric tilings, higher dimensional analogs, polyomino type problems on other regular lattice grids (triangular, hexagonal), etc. However, the problem is NP-hard in general, and we can give definite answers only in a limited number of cases.

This enthralling problem from recreational mathematics has attracted attention of both mathematicians and non-experts. There were many results establishing criteria for proper tilings by some specific polyomino shapes (see **[9]**, **[10]**, **[11]**, **[18]** and **[19]**). Conway and Lagarias developed in **[5]** the so-called 'boundary-word method' for addressing this question. Their ideas were further developed by Reid in **[17]** who assigned to each set of tiles $\mathcal{T}$ *the homology and the homotopy group of tilings* and formulated a necessary condition for existence of a proper tiling of a finite region M in a plane.

Reid's powerful idea allows natural generalization to a much bigger class of combinatorial tilings. Instead of considering planar regions, we study regions which are obtained by identifying parts of the boundary of a planar region resulting in a flat Riemann surface. The only flat compact Riemann surfaces are the torus and Klein bottle, but one can give higher-genus surfaces a flat metric everywhere except at certain cone points, and then remove neighborhoods of the singular points to get a flat surface with boundary. Surfaces with a flat metric obtained by pairwise identification of sides of a collection of plane polygons via translations of their sides, are called translation surfaces. Translation surfaces can also be defined as Riemann surfaces with a holomorphic 1-form. In particular, we are interested in a subclass of translation surfaces called a square-tiled surface. A square-tiled surface is any translation surface obtained from a polygon P which is itself obtained by putting a collection of copies of the unit square side by side. In general, the total angle around a corner of a square of a square-tiled surface S is a non-trivial multiple of 2π. Any such point is called a conical singularity of S. In this paper, we study the problem of tiling a surface S subdivided into a finite 'combinatorial' grid by a finite set of polyomino shapes T and define the homology group $H_S(\mathcal{T})$.

Square-tiled and translation surfaces arise in dynamical systems, where they can be used to model billiards, and in Teichmüller theory. They have a rich mathematical structure and may be studied from multiple points of view (flat geometry, algebraic geometry, combinatorial group theory, etc.). We present some new results and illustrate examples explaining the application of the homology group of generalized polyomino type tilings in the combinatorial and the topological context.

In Section 2, we introduce the homology tiling group for finite square grids on surfaces with boundaries based on **[17]**. Several results about the impossibility of tiling certain concrete square-tiled surfaces are proved using the homology group of the tiling in Section 3. Our main novelty lies in Theorem 12 which establishes a general result connecting the I-polyomino shape with the genus of the surface.

2. TILING PROBLEM ON SURFACES

The standard square grid in the plane is characterized by the property that exactly four edges meet at each vertex, and each vertex is shared by four squares in the grid. We assume that every edge in a combinatorial grid on a surface is shared by exactly two squares, unless it is on the boundary. This local property allows us to define a polyomino tiling on a topological surface in the same way as in the planar case, and we will refer to such a structure as *the square grid on a surface*. For example, identification of parallel edges of the boundary of $m \times n$ grid in the same directions provides such a grid on torus. Identification of two pairs of parallel sides of an $m \times m$, $m \geq 3$ square, but in the opposite direction in one of the pairs, provides examples of square grids on Klein bottle, see Figure 4. However, if the surface has no boundary, then each vertex is shared by four squares, so the number of vertices equals the number of squares. Likewise, each edge is shared by two squares, so the number of edges is twice the number of squares. This makes the Euler characteristic

$$\chi(M) = V - E + F = F - 2F + F = 0,$$

so M is either the torus or the Klein bottle.

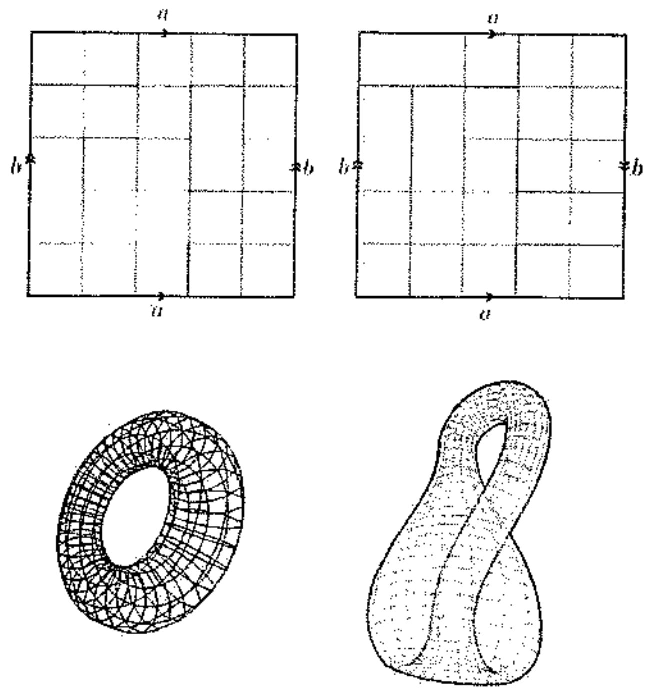

Figure 4: Square grids on a torus and a Klein bottle

On topological surfaces with boundaries, square grids are not rare structures. One way to obtain them is by identification of certain faces of a finite region in a planar square grid. Identification of faces allows the additional possibility for placing a polyomino tile, so we have to develop means to treat tiling problems. Surfaces obtained by gluing sides of a polygon are extensively studied in mathematics and this is an interesting research topic in itself (see [1], [12], [13] and [21]).

Actually, above mentioned combinatorial structures are directly related to mathematical concepts known as *translation surfaces*. Combinatorially, a translation surface may be defined in the following way. Let $P_1, \ldots, P_m$ be a collection of polygons in the Euclidean plane and suppose that for every side s_i of any P_k there is a side s_j of some P_l with $j \neq i$ and $s_j = s_i + \vec{v}_i$ for some nonzero vector $\vec{v}_i$ and so that $\vec{v}_j = -\vec{v}_i$. The space obtained by identifying all s_i with their corresponding s_j through the map $x \mapsto x + \vec{v}_i$ is a translation surface.

A particular class of translation surfaces known as square-tiled surfaces is of wide interest for mathematics. A square-tiled surface is an orientable connected surface obtained from a finite collection of unit squares in a plane after identifications of pairs of parallel sides via adequate translations. In general, the total angle around a corner of a square of a square-tiled surface M is a non-trivial multiple of 2π and any such point is called a conical singularity of M. In our considerations we will consider flat surfaces with cone points with cone angle a multiple of $\frac{\pi}{2}$.

The tiling problem for a finite subset of the regular planar square grid by a finite set of polyomino prototiles has been studied extensively in the past few decades. However, there exist many other topological 2-manifolds which admit subdivision into a finite number of squares which preserves the structure of regular square grid and for which the tiling problem is also defined. One natural way to obtain such structures is by gluing some of the faces of a finite subset of regular square lattice in the plane, and some results and examples of polyomino tiling problems in this context are known in literature under the notion of topological tilings. Special cases of cylinders, torus, Möbius strip, Klein bottle and projective plane with a 2-disk removed were studied in [8], [20] and [14].

Several techniques for finding obstructions to tiling are known, and one of the most charming is that of a 'generalized chessboard coloring'. This method rests on the fact that the chessboard with two opposite square corners removed cannot be tiled by dominoes, as the difference between the number of white and black squares is two, see [7]. The general idea is to use several colors and color the squares of the considered region in a special pattern 'sensitive' to the given set of polyominoes. In other words, the coloring imposes some number theoretical condition which serves as an obstruction to a tiling. However, it is not easy to find a coloring argument for proving nonexistence of a tiling. Michael Reid introduced in [17] the so-called homology group of a tiling and showed that proof of nontriviality of a special element in this group assigned to the finite subset of regular square lattice produces a generalized chessboard coloring argument. His homology tiling group method is therefore at least as powerful as the coloring argument. In the same paper, Reid gave many examples where the tiling homology group is inefficient for proving non-existence of a tiling.

The problem of polyomino tilings was studied by Conway and Lagarias in [5] where they introduced a new technique using boundary word invariants to formulate necessary conditions for the existence of tilings. Based on their ideas, Reid presented in **[17]** a new strategy for treating tiling problems, working with the so-called homotopy group of tiling. Reid's *homotopy tiling group method* was so far the most successful in establishing necessary criteria for existence of tilings.

Our main observation is that Reid's tiling homology group method can be applied to studying topological tilings. A standard model for obtaining topological surfaces is identification of sides of a polygon and as clearly presented in **[13]** and **[21]**.

Let M be a topological surface with boundary obtained by gluing of sides of some finite subset R of the regular square grid in the plane and let $\mathcal{T}$ be a finite set of polyomino tiles. Gluing of faces provides more ways for placement of tiles from $\mathcal{T}$ on M then in the case of R, so M may be tiled even if R does not admit a tiling by tiles from $\mathcal{T}$. We introduce the tiling homology group $H(M, \mathcal{T})$ in the same fashion as Michael Reid.

Let A be the free abelian group generated by the set of cells of M. We assume that all cells of M preserve labeling by (i, j) from R. The generator of A corresponding to the cell (i, j) is denoted by $a_{i,j}$. Let $B(M, \mathcal{T})$ be the subgroup generated by elements corresponding to all possible placements of tiles in $\mathcal{T}$, i.e. by the sums of elements assigned to cells of M that can be covered by a tile from $\mathcal{T}$.

Definition 1. *The tiling homology group of $(M, \mathcal{T})$ is the quotient group*

$$H(M, \mathcal{T}) = A/B(M, \mathcal{T}).$$

Let us denote by $\bar{a}_{i,j}$ the image of $a_{i,j}$ in $H(M, \mathcal{T})$. As in the planar case, there is an element $\Theta \in H(M, \mathcal{T})$ assigned to M

$$\Theta := \sum_{(i,j) \in M} \bar{a}_{i,j}$$

which is clearly zero when there is a tiling of M by polyominoes from $\mathcal{T}$. Thus, Θ is an obstruction to tiling. Recall that Reid considered in his paper the so-called *signed tiling*, where he allowed polyomino tiles to have positive and negative signs. Clearly, the signed tiling of M by $\mathcal{T}$ exists if and only if Θ is trivial in $H(M, \mathcal{T})$.

Reid's [**17**, Proposition 2.10] also holds for topological tilings by polyominoes. It states that nontrivial Θ produces special numbering of cells in M that yields a generalized chessboard coloring argument. We adapt his proof to the case of topological tilings.

Proposition 2. *Let M be a topological surface with boundary with a finite square grid and finite set of polyominoes $\mathcal{T}$ such that Θ is nontrivial in $H(M, \mathcal{T})$. Then there is the numbering of the cells in M by rational numbers such that*

i) for any placement of a tile from $\mathcal{T}$, the total sum of covered numbers is an integer; and *ii) the total covered by the cells of M is not an integer.*

Proof. Consider the cyclic subgroup $\langle\Theta\rangle \subseteq H(\mathcal{T})$ generated by Θ. We define a homomorphism $\varphi : \langle\Theta\rangle \to \mathbb{Q}/\mathbb{Z}$ with $\varphi(\Theta) \neq 0$. If Θ has infinite order we set $\varphi(\Theta) = \frac{1}{2} \mod \mathbb{Z}$, while if Θ has finite order $n > 1$, then we define $\varphi(\Theta) = \frac{1}{n} \mod \mathbb{Z}$. Since $\mathbb{Q}/\mathbb{Z}$ is a divisible abelian group, the homomorphism φ extends to a homomorphism $H(M,\mathcal{T}) \to \mathbb{Q}/\mathbb{Z}$, also called φ. Here we used the familiar fact about equivalence of the notions of injective group and divisible group for abelian groups [4, Proposition 6.2]. Since A is a free abelian group, the composite map

$$A \longrightarrow A/B(M,\mathcal{T}) = H(M,\mathcal{T}) \xrightarrow{\varphi} \mathbb{Q}/\mathbb{Z}$$

lifts to a homomorphism $\psi : A \to \mathbb{Q}$, such that the following diagram commutes

$$\begin{array}{ccc} A & \xrightarrow{\psi} & \mathbb{Q} \\ \downarrow & & \downarrow \\ H(M,\mathcal{T}) & \xrightarrow{\varphi} & \mathbb{Q}/\mathbb{Z} \end{array}$$

where the vertical surjections are the quotient maps. Desired numbering of the cells is defined by ψ, and since $B(M,\mathcal{T})$ is in the kernel of $A \to \mathbb{Q}/\mathbb{Z}$, every tile placement covers an integral total. But, $\varphi(\Theta) \neq 0$ and total of the cells in M is not an integer. □

Reid's tiling homology group was systematically studied using Gröbner bases in the works of Muzika-Dizdarević, Timotijević and Živaljević, see **[15]** and **[16]**.

3. NONEXISTENCE OF POLYOMINO TILINGS ON SURFACES

In this section we prove several results on nonexistence of tilings on surfaces of different genus with boundaries by some given polyomino sets as an illustration of the homology method.

First we formulate three results for polyomino tilings on a torus square grid. Such cases were also studied in the past **[20]** as they are close to the planar case.

Theorem 3. *A square torus grid of dimension $(4m+2) \times (4n+2)$ cannot be tiled by I-tetrominoes, see Figure 2.*

Proof. Consider a $(4m+2) \times (4n+2)$ square torus grid model in a plane with cells labelled as in Figure 3.1.

Investigate all possible placements of a tile in the given model. To each placement one can assign one of two types of relations:

$$\bar{a}_{i,j} + \bar{a}_{i,j+1} + \bar{a}_{i,j+2} + \bar{a}_{i,j+3} = 0 \quad \text{and} \quad \bar{a}_{i,j} + \bar{a}_{i+1,j} + \bar{a}_{i+2,j} + \bar{a}_{i+3,j} = 0$$

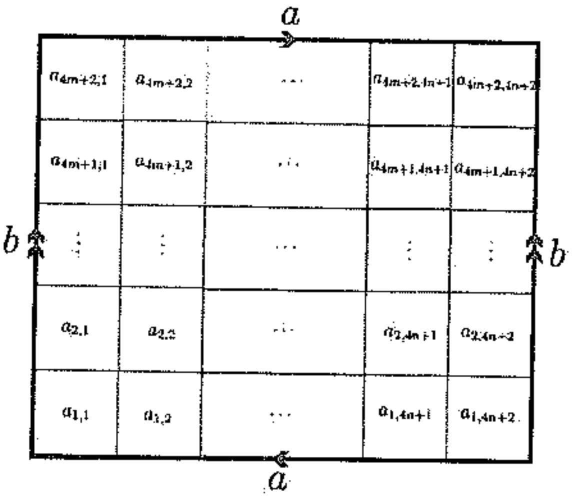

Figure 3.1: Torus grid of dimension $(4m+2)\times(4n+2)$

where $i=1,\dots,4m+2$ labels a row, and $j=1,\dots,4n+2$ labels a column on the given torus grid. We assume that indices of rows is modulo $4m+2$ and modulo $4n+2$ for columns in the relations above. Considering the relation

$$\bar{a}_{i,j+1}+\bar{a}_{i,j+2}+\bar{a}_{i,j+3}+\bar{a}_{i,j+4}=0$$

we obtain that in the homology group of this tiling it holds that

$$\bar{a}_{i,j}=\bar{a}_{i,j+4}$$

for all $i,j\in\{1,2,\dots,4k+2\}$. Analogously, $\bar{a}_{i,j}=\bar{a}_{i+4,j}$.

From the relations corresponding to placements over the identified faces of the rectangle representing our torus grid, we obtain additional cells of the grid whose corresponding generators in the homology group of tiling are equal. Using

$$\bar{a}_{i,4m-1}+\bar{a}_{i,4m}+\bar{a}_{i,4m+1}+\bar{a}_{i,4m+2}=0 \quad \text{and}$$
$$\bar{a}_{i,4m}+\bar{a}_{i,4m+1}+\bar{a}_{i,4m+2}+\bar{a}_{i,1}=0$$

we conclude that $\bar{a}_{i,1}=\bar{a}_{i,4m-1}$. In the same fashion we deduce that $\bar{a}_{i,2}=\bar{a}_{i,4m}$, $\bar{a}_{1,i}=\bar{a}_{4n-1,i}$ and $\bar{a}_{2,i}=\bar{a}_{4n,i}$ for all i. Combining the equalities above, we obtain

$$\bar{a}_{i,j}=\begin{cases}\bar{a}_{1,1}, & \text{if } i\equiv 1 \pmod 2,\ j\equiv 1 \pmod 2,\\ \bar{a}_{1,2}, & \text{if } i\equiv 1 \pmod 2,\ j\equiv 0 \pmod 2,\\ \bar{a}_{2,1}, & \text{if } i\equiv 0 \pmod 2,\ j\equiv 1 \pmod 2,\\ \bar{a}_{2,2}, & \text{if } i\equiv 0 \pmod 2,\ j\equiv 0 \pmod 2.\end{cases}$$

as depicted in Figure 3.2.

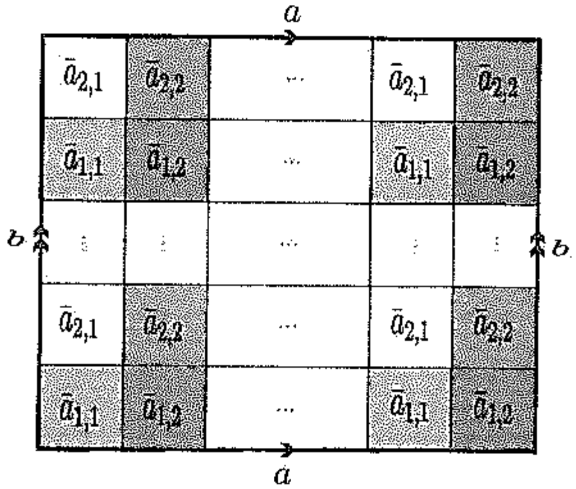

Figure 3.2: Coloring of the equivalent cells of the torus grid

If we put I-tetromino shape on the torus grid with equivalent cells we obtain one of the following relations

$$2\bar{a}_{1,1} + 2\bar{a}_{1,2} = 0, \qquad 2\bar{a}_{1,1} + 2\bar{a}_{2,1} = 0,$$
$$2\bar{a}_{2,1} + 2\bar{a}_{2,2} = 0, \qquad 2\bar{a}_{1,2} + 2\bar{a}_{2,2} = 0.$$

Therefore, our homology group is isomorphic to the quotient group of the free abelian group with four generators by the four relations given above. Let us observe that one of these relations can be obtained from the remaining three so we can omit the relation $2\bar{a}_{2,1} + 2\bar{a}_{2,2} = 0$. We can consider the presentation of the group using the following four generators $a = \bar{a}_{1,1}$, $b = \bar{a}_{1,1} + \bar{a}_{1,2}$, $c = \bar{a}_{1,1} + \bar{a}_{2,1}$ and $d = \bar{a}_{2,2} - \bar{a}_{1,1}$. It is clear that $2b = 2c = 0$, and little more effort gives $2d = 0$. Thus, our homology group of tiling is isomorphic to

$$G(a, b, c, d | 2b = 2c = 2d = 0) \cong \mathbb{Z} \oplus (\mathbb{Z}_2)^3.$$

It is easily seen that everything but the top two (or bottom two) rows of our grid are easily tiled by vertical I-tetrominoes, and that in the top two rows everything but the right-most two columns are tiled by horizontal I-tetrominoes, so Θ is the sum of elements corresponding to the four upper right cells. Thus,

$$\Theta = \bar{a}_{1,1} + \bar{a}_{1,2} + \bar{a}_{2,1} + \bar{a}_{2,2} = b + c + d$$

is nontrivial in the tiling homology group, so desired tiling is not possible. □

Remark 4. *We can reach the same conclusion using coloring of the square torus grid as in Figure 3.2. Each tile covers 2 blue and 2 yellow cells, or 2 blue and 2* *red, or 2 yellow and 2 green, or 2 red and 2 green. Since the number of cells of each color is odd and each tile covers an even number of cells of the same color, we conclude that tiling is not possible.*

Theorem 5. *A square torus grid of dimension $(4m+2)\times(4n+2)$ cannot be tiled with T tetrominoes.*

Proof. Consider the torus grid presented as in Figure 3.1. Consider all possible placements of T tetromino. To each placement we can assign one of the following relations:

$$\begin{aligned}
\bar{a}_{i,j}+\bar{a}_{i,j+1}+\bar{a}_{i,j+2}+\bar{a}_{i+1,j+1} &= 0,\\
\bar{a}_{i,j}+\bar{a}_{i,j+1}+\bar{a}_{i,j+2}+\bar{a}_{i-1,j+1} &= 0,\\
\bar{a}_{i,j}+\bar{a}_{i+1,j}+\bar{a}_{i+2,j}+\bar{a}_{i+1,j+1} &= 0 \quad \text{and}\\
\bar{a}_{i,j}+\bar{a}_{i+1,j}+\bar{a}_{i+2,j}+\bar{a}_{i+1,j-1} &= 0,
\end{aligned}$$

where we use the same labelling as in the proof of Theorem 3. From them we directly deduce that in the homology group of tiling it holds that

$$\bar{a}_{i+2,j}=\bar{a}_{i,j}=\bar{a}_{i,j+2}$$

for all i and j. Therefore,

$$\bar{a}_{i,j}=\begin{cases}\bar{a}_{1,1}, & \text{if } \; i-j\equiv 0 \pmod 2;\\ \bar{a}_{1,2}, & \text{if } \; i-j\equiv 1 \pmod 2,\end{cases}$$

as it is illustrated in Figure 5.1.

Placing a T-tetromino shape on the torus grid with equivalent cells, we get one of the following two relations

$$\begin{aligned}
3\bar{a}_{1,1}+\bar{a}_{1,2} &= 0 \quad \text{and}\\
3\bar{a}_{1,2}+\bar{a}_{1,1} &= 0.
\end{aligned}$$

Therefore, our homology group is isomorphic to the group

$$G(\bar{a}_{1,1}|8\bar{a}_{1,1}=0)\cong\mathbb{Z}_8.$$

Our grid has $2m$ cells $\bar{a}_{1,1}$ and $\bar{a}_{1,2}$, where $k=(2m+1)(2n+1)$. So the element that corresponds to this grid

$$\Theta \;=\; 2k\bar{a}_{1,1}+2k\bar{a}_{1,2}=-4k\bar{a}_{1,1}=4\bar{a}_{1,1}$$

is nontrivial in the homology group, so desired tiling does not exist. □

Remark 6. *The same conclusion can be obtained using coloring in Figure 5.1 and parity argument for the total number of cells in the grid.*

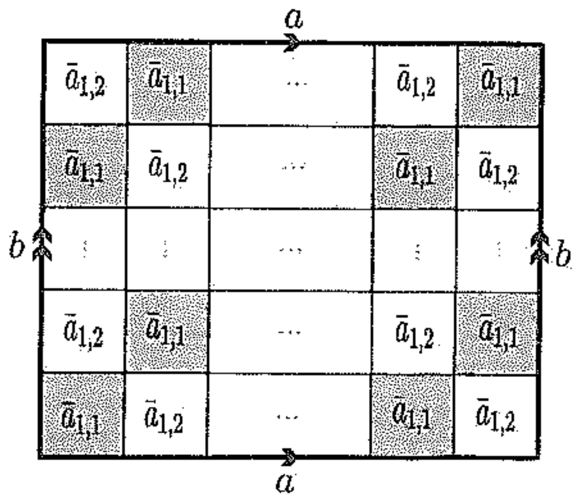

Figure 5.1: Coloring of the equivalent cells of the torus grid

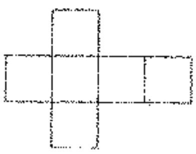

Figure 7.1: X hexomino

Theorem 7. *A square torus grid of dimension $(4m+2)\times(4n+2)$ cannot be tiled with X hexominoes (Figure 7.1).*

Proof. Consider planar model of torus grid of dimension $(4m+2)\times(4n+2)$ as in Figure 3.1. Examine all possible horizontal placements of our tile. Each of them yields a relation

$$\bar{a}_{i,j}+\bar{a}_{i,j+1}+\bar{a}_{i,j+2}+\bar{a}_{i,j+3}+\bar{a}_{i+1,j+1}+\bar{a}_{i-1,j+1}=0, \tag{7.1}$$

where where the rows and columns are labelled analogously as in the proof of Theorem 3. From (7.1) we conclude that in the homology group of this tilling it holds $\bar{a}_{i,j}=\bar{a}_{i,j+4}$ for all i and j. Since $\bar{a}_{i,4n-1}=\bar{a}_{i,1}$, $\bar{a}_{i,4n}=\bar{a}_{i,2}$, $\bar{a}_{i,4n+1}=\bar{a}_{i,3}$ and $\bar{a}_{i,4n+2}=\bar{a}_{i,4}$ we further get that for all i and j it also holds $\bar{a}_{i,j}=\bar{a}_{i,j+2}$.

Analogous consideration of vertical placements implies $\bar{a}_{i,j}=\bar{a}_{i+2,j}$ for all i and j. Equivalences of the cells in the grid in the homology group of tiling are depicted in Figure 7.2.

Thus, we deduce that the homology group of tiling is the quotient of the free

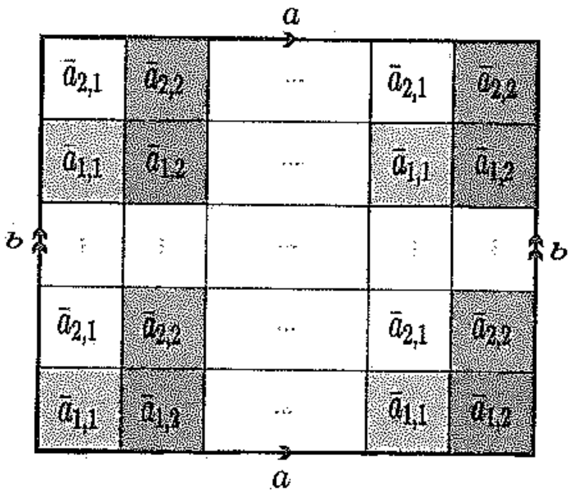

Figure 7.2: Coloring of the equivalent cells of the torus grid

abelian group with four generators $G\langle \bar{a}_{1,1}, \bar{a}_{1,2}, \bar{a}_{2,1}, \bar{a}_{2,2}\rangle$ modulo following relations

$$\begin{aligned}
2\bar{a}_{1,2} + 2\bar{a}_{2,1} + 2\bar{a}_{2,2} &= 0,\\
2\bar{a}_{1,1} + 2\bar{a}_{2,1} + 2\bar{a}_{2,2} &= 0,\\
2\bar{a}_{1,1} + 2\bar{a}_{1,2} + 2\bar{a}_{2,2} &= 0 \quad \text{and}\\
2\bar{a}_{1,1} + 2\bar{a}_{1,2} + 2\bar{a}_{2,1} &= 0.
\end{aligned}$$

We consider the presentation of the homology group of tiling using the following generators $x = \bar{a}_{1,2} + \bar{a}_{2,1} + \bar{a}_{2,2}$, $y = \bar{a}_{1,1} + \bar{a}_{2,1} + \bar{a}_{2,2}$, $z = \bar{a}_{1,1} + \bar{a}_{1,2} + \bar{a}_{2,2}$ and $t = \bar{a}_{1,1} + \bar{a}_{1,2} + \bar{a}_{2,1} + \bar{a}_{2,2}$. The upper relations in new generators are

$$2x = 2y = 2z = 6t - 2x - 2y - 2z = 0.$$

Finally, we find that the homology group of tiling is

$$G\langle x, y, z, t \mid 2c = 2b = 2z = 6t = 0\rangle \cong (\mathbb{Z}_2)^3 \oplus \mathbb{Z}_6.$$

It follows that the element corresponding to this grid

$$\begin{aligned}
\Theta &= (2k+1)\bar{a}_{1,1} + (2k+1)\bar{a}_{1,2} + (2k+1)\bar{a}_{2,1} + (2k+1)\bar{a}_{2,2}\\
&= (2k+1)(\bar{a}_{1,1} + \bar{a}_{1,2} + \bar{a}_{2,1} + \bar{a}_{2,2})\\
&= (2k+1)u
\end{aligned}$$

is a nontrivial element in the homology group as 6 does not divide $2k+1$. Therefore, tiling does not exist. □

Remark 8. *The same conclusion can be obtained by colouring of torus grid as in Figure 7.2. Each tile covers 2 blue cells, 2 red and 2 green or 2 blue, 2 yellow and 2 green or 2 red, 2 yellow and 2 green or 2 blue, 2 red and 2 green cells. Given that the number of cells of each color is odd, and every tile covers even number of cells of each color, we conclude that tilling is not possible.*

Now we prove some results on surfaces with boundaries. As we will see, topology contributes significantly to the homology group of tiling.

Theorem 9. *A square grid on a non-orientable surface of genus 6 with boundary formed by identifying the sides of a dodecagon consisting of five $4k \times 4k$ squares and removing 20 corner cells around cone point as in Figure 9.1 cannot be tiled with I-tetrominoes and Z-tetrominoes.*

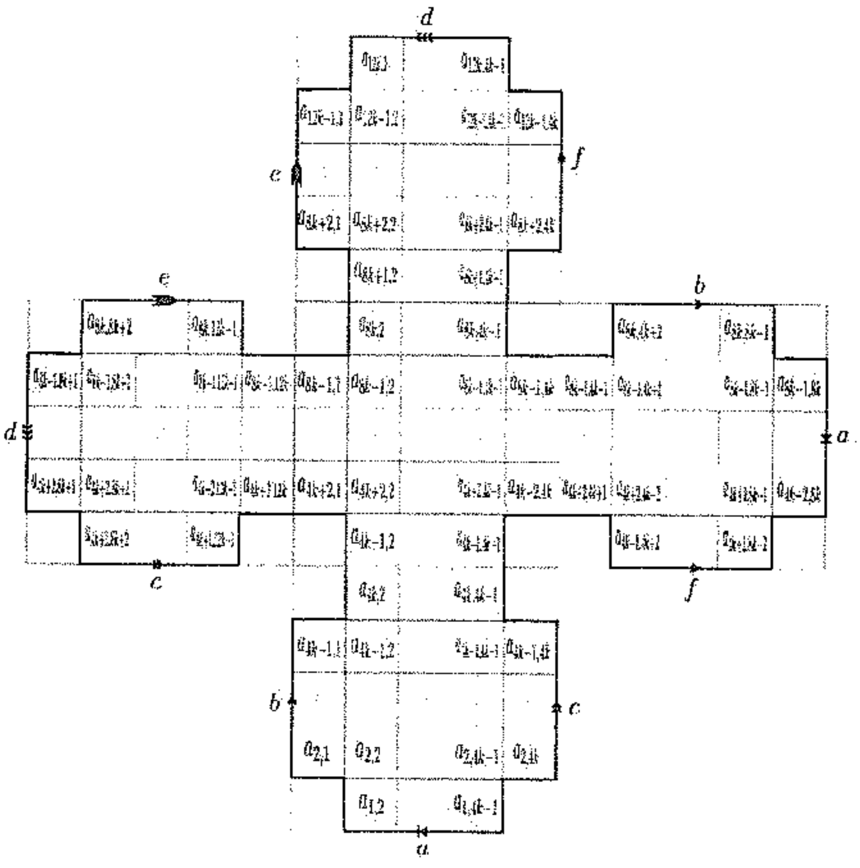

Figure 9.1: Square grid on a non-orientable surface of genus 6 with boundary

Proof. Let us denote the cells of this square grid as in Figure 9.1. Observe that cells $a_{1,1}$, $a_{1,4k}$, $a_{4k,1}$, $a_{4k,4k}$, $a_{4k+1,8k+1}$, $a_{4k+1,12k}$, $a_{4k+1,1}$, $a_{4k+1,4k}$, $a_{4k+1,4k+1}$, $a_{4k+1,8k}$, $a_{8k,8k+1}$, $a_{8k,12k}$, $a_{8k,1}$, $a_{8k,4k}$, $a_{8k,4k+1}$, $a_{8k,8k}$, $a_{8k+1,1}$, $a_{8k+1,4k}$, $a_{12k,1}$ and $a_{12k,4k}$ are deleted and that, topologically, after gluing their union becomes a disk. Thus, we study a gluing of non-orientable surface of genus 6 with one boundary component.

Using I-tetrominoes it is easy to deduce that in the homology group of tiling it holds that $\bar{a}_{i,j} = \bar{a}_{i+4,j}$ and $\bar{a}_{i,j} = \bar{a}_{i+4,j}$.

A placement of a Z-tetromino yields one of the following two relations

$$\bar{a}_{i,j}+\bar{a}_{i+1,j}+\bar{a}_{i+1,j+1}+\bar{a}_{i+2,j+1} = 0 \quad \text{and} \quad \bar{a}_{i+1,j}+\bar{a}_{i+1,j+1}+\bar{a}_{i+2,j+1}+\bar{a}_{i+2,j+2} = 0.$$

They imply $\bar{a}_{i+2,j+2} = \bar{a}_{i,j}$.

Considering placement of I-tetromino across the edge d it is easy to see that $\bar{a}_{4k+2,8k+1} = \bar{a}_{12k-3,2}$, $\bar{a}_{4k+2,8k+2} = \bar{a}_{12k-2,2}$, $\bar{a}_{4k+2,8k+3} = \bar{a}_{12k-1,2}$ and $\bar{a}_{4k+2,8k+4} = \bar{a}_{12k,2}$. With the relations above we obtain the following equivalences in the homology group of this tiling depicted in Figure 9.2.

Thus, the homology group of tiling is a free abelian group with four generators $\bar{a}_{1,2}, \bar{a}_{1,3}, \bar{a}_{2,3}, \bar{a}_{2,4}$ quotiented by the following relations

$$\begin{aligned}
\bar{a}_{1,2} + \bar{a}_{1,3} + \bar{a}_{2,3} + \bar{a}_{2,4} &= 0, \\
2\bar{a}_{1,2} + 2\bar{a}_{1,3} &= 0, \\
2\bar{a}_{1,3} + 2\bar{a}_{2,3} &= 0, \\
2\bar{a}_{2,3} + 2\bar{a}_{2,4} &= 0, \\
2\bar{a}_{1,2} + 2\bar{a}_{2,4} &= 0.
\end{aligned}$$

We eliminate generator $\bar{a}_{2,4}$ from its presentation and consider generators $\bar{a}_{1,3}$, $b = \bar{a}_{1,2} + \bar{a}_{1,3}$ and $c = \bar{a}_{1,3} + \bar{a}_{2,3}$. We obtain that our group of homology is isomorphic to $\mathbb{Z} \oplus \mathbb{Z}_2^2$.

Our square grid contains $20k^2$ cells $\bar{a}_{1,2}$, $5(4k^2-1)$ cells $\bar{a}_{1,3}$ and $\bar{a}_{2,4}$, as well as $10(2k^2-1)$ cells $\bar{a}_{2,3}$. The element corresponding to this grid

$$\begin{aligned}
\Theta &= 20k^2\bar{a}_{1,2} + 5(4k^2-1)\bar{a}_{1,3} + 10(2k^2-1)\bar{a}_{2,3} + 5(4k^2-1)\bar{a}_{2,4} \\
&= 20k^2(\bar{a}_{1,2} + \bar{a}_{1,3} + \bar{a}_{2,3} + \bar{a}_{2,4}) - 5\bar{a}_{1,3} - 10\bar{a}_{2,3} - 5\bar{a}_{2,4} \\
&= 5\bar{a}_{1,2} - 5\bar{a}_{1,3} = b - 10\bar{a}_{1,3}
\end{aligned}$$

is a non-trivial element of the homology group and desired tiling is not possible. □

Theorem 10. *A grid on a non-orientable surface of genus 4 with boundary is formed by identifying the sides of a dodecagon consisting of five $4k \times 4k$ squares and with removed 20 cells around cone points as in Figure 10.1 cannot be tiled with L-tetrominoes.*

Proof. Model in Figure 10.1 after gluing along marked sides and deletion of 20 corner cells gives a non-orientable surface of genus 4 with three boundary components. Denote the cells in the grid as in the previous example.

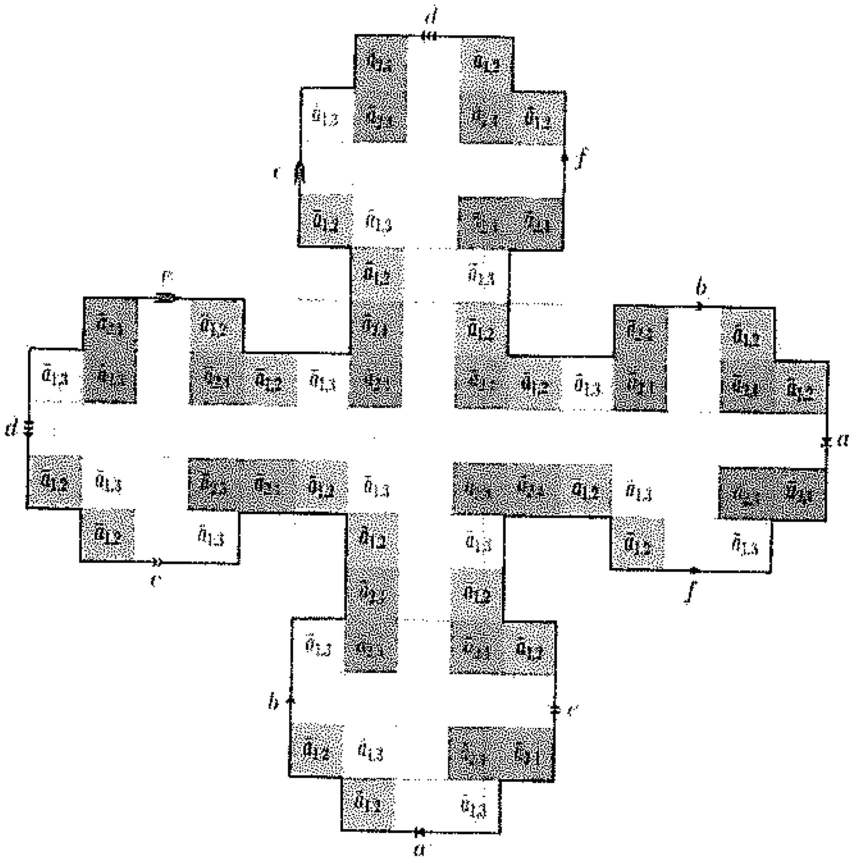

Figure 9.2: Coloring of the equivalent cells in square grid on a non-orientable surface of genus 6 with boundary

Placing L-tetromino in the given model in vertical position before taking identification into account will give one of the two relations

$$\tilde{a}_{i,j} + \tilde{a}_{i+1,j} + \tilde{a}_{i+2,j} + \tilde{a}_{i+2,j-1} = 0 \quad \text{and} \tag{10.1}$$

$$\tilde{a}_{i,j} + \tilde{a}_{i+1,j} + \tilde{a}_{i+2,j} + \tilde{a}_{i+2,j+1} = 0 \tag{10.2}$$

in the homology group of tiling. From (10.1) and (10.2) we obtain that in the group of homology of this tiling the cells $\tilde{a}_{i,j-1} = \tilde{a}_{i,j+1}$ are equivalent. Analogously, it holds that $\tilde{a}_{i-1,j} = \tilde{a}_{i+1,j}$ are equivalent in the homology group of this tiling.

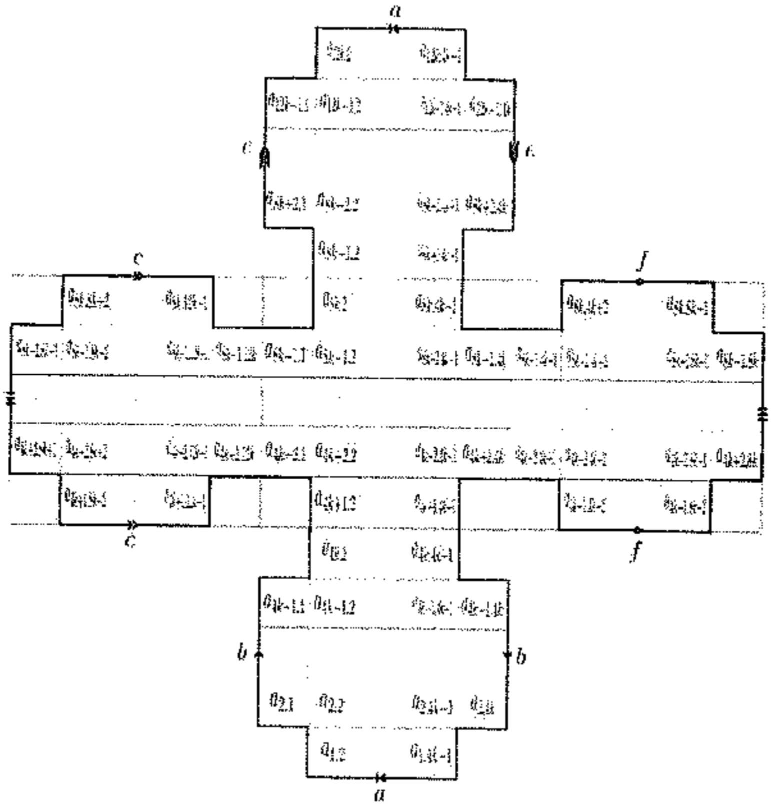

Figure 10.1: Square grid on a non-orientable surface of genus 4 with three boundary components

We summarize all upper equivalences of cells in

$$\tilde{a}_{i,j} = \begin{cases} \tilde{a}_{1,1}, & \text{if} \quad i \equiv 1 \pmod 2,\ j \equiv 1 \pmod 2, \\ \tilde{a}_{1,2}, & \text{if} \quad i \equiv 1 \pmod 2,\ j \equiv 0 \pmod 2, \\ \tilde{a}_{2,1}, & \text{if} \quad i \equiv 0 \pmod 2,\ j \equiv 1 \pmod 2, \\ \tilde{a}_{2,2}, & \text{if} \quad i \equiv 0 \pmod 2,\ j \equiv 0 \pmod 2. \end{cases}$$

Consider a placement of L-tetromino along edge denoted by e in Figure 10.1 and corresponding equations in the homology group of tiling

$$\begin{aligned} \tilde{a}_{1,1} + \tilde{a}_{1,2} + \tilde{a}_{2,1} + \tilde{a}_{1,2} &= 0 \quad \text{and} \\ \tilde{a}_{2,1} + \tilde{a}_{1,2} + \tilde{a}_{2,1} + \tilde{a}_{1,2} &= 0. \end{aligned}$$

From them we deduce that $\tilde{a}_{1,1} = \tilde{a}_{2,1}$. In a similar way we obtain that $\tilde{a}_{1,2} = \tilde{a}_{2,2}$. These equivalences are illustrated in Figure 10.2.

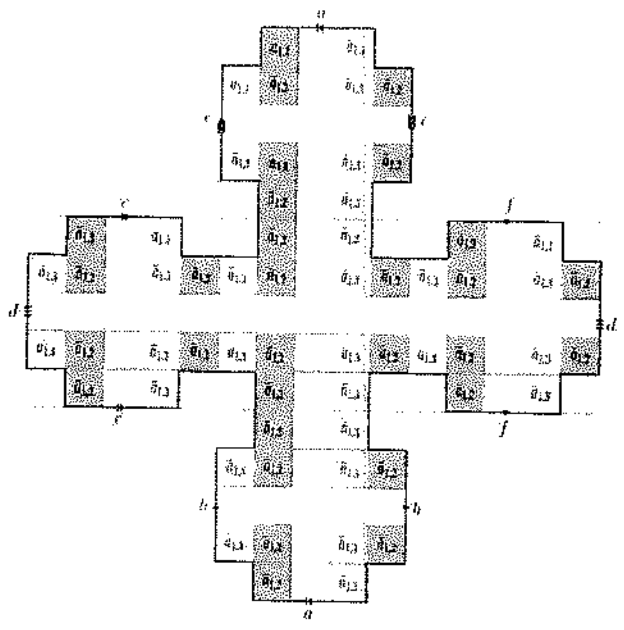

Figure 10.2: Equivalent cells on square grid on a non-orientable surface of genus 4 with boundary

Placement of L tetromino on the grid with equivalent cells, including placements across glued sides, we obtain one of the two relations

$$\begin{aligned} 3\bar{a}_{1,1} + \bar{a}_{1,2} &= 0 \quad \text{and} \\ 3\bar{a}_{1,2} + \bar{a}_{1,1} &= 0. \end{aligned}$$

Now we conclude that $8\bar{a}_{1,1} = 0$. Therefore, the homology group is isomorphic to the group

$$G\langle \bar{a}_{1,1} | 8\bar{a}_{1,1} = 0 \rangle \cong \mathbb{Z}_8.$$

Our square grid contains $10(4k^2 - 1)$ cells $a_{1,1}$ and $a_{1,2}$, so the element assigned to this grid

$$\Theta = 10(4k^2 - 1)\bar{a}_{1,1} + 10(4k^2 - 1)\bar{a}_{1,2} = 4\bar{a}_{1,1}$$

is a non-trivial element in the homology group of tiling and it is not possible to tile the given grid using L-tetrominoes. □

Theorem 11. *A square grid on an orientable surface of genus 3 with boundary formed by identifying the sides of a dodecagon consisting of five $4k \times 4k$ squares and removing 20 cells meeting in the cone point as in Figure 11.1 cannot be tiled by T-tetrominoes.*

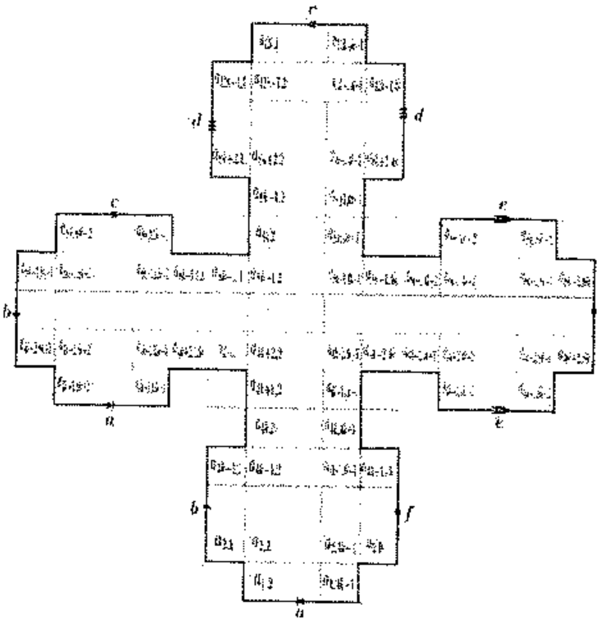

Figure 11.1: Square grid on an orientable genus 3 surface with boundary

Proof.

It is straightforward to check that model in Figure 11.1 after gluing along marked sides and deletion of 20 corner cells gives a genus 3 surface with one boundary component. Denote the cells in the grid as in the previous theorem. The following equality is easily obtained

$$\bar{a}_{i,j} = \begin{cases} \bar{a}_{1,1}, & \text{if} \quad i-j \equiv 0 \pmod 2, \\ \bar{a}_{1,2}, & \text{if} \quad i-j \equiv 1 \pmod 2, \end{cases}$$

as it is illustrated in Figure 11.2.

If we put T tetrominoes on the grid with equivalent cells, even placing it across a glued sides, we obtain one of the two relations

$$\begin{aligned} 3\bar{a}_{1,3} + \bar{a}_{1,2} &= 0 \quad \text{and} \\ 3\bar{a}_{1,2} + \bar{a}_{1,3} &= 0. \end{aligned}$$

Therefore, we get that the homology group of this is isomorphic to the group

$$G\langle \bar{a}_{1,2} | 8\bar{a}_{1,2} = 0 \rangle \cong \mathbb{Z}_8.$$

Our square grid contains $10(4k^2-1)$ cells $\bar{a}_{1,2}$ and $10(4k^2-1)$ cells $\bar{a}_{1,3}$, so

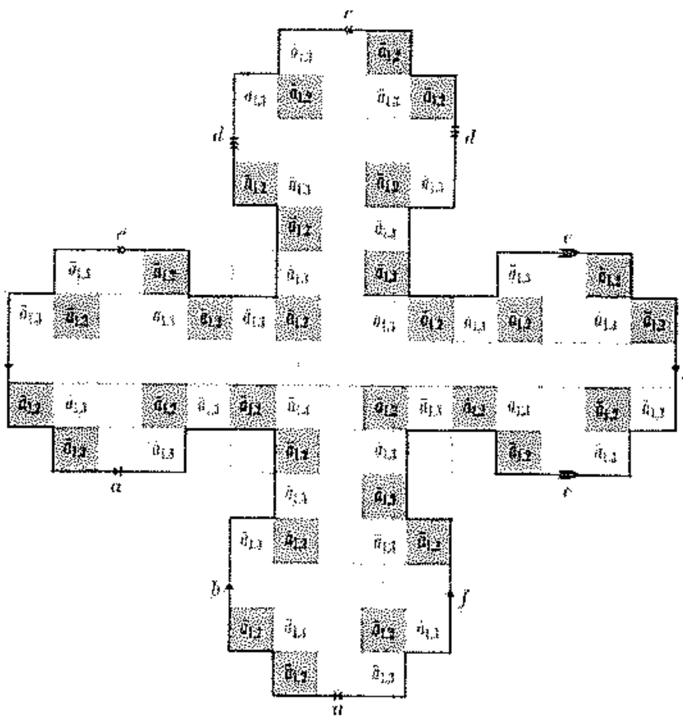

Figure 11.2: Equivalent cells on square grid on an orientable genus 3 surface with boundary

the element assigned to this grid is

$$\begin{aligned}\Theta &= 10(4k^2-1)\bar{a}_{1,2} + 10(4k^2-1)\bar{a}_{1,3} \\ &= 40k^2\bar{a}_{1,2} - 10\bar{a}_{1,2} - 120k^2\bar{a}_{1,2} + 30\bar{a}_{1,2} = 4\bar{a}_{1,2}.\end{aligned}$$

Θ is a non trivial element in the homology group of tiling, and therefore it is not possible to til the given square grid using T-tetrominoes. □

Theorem 12. *A square grid on an orientable surface of genus $2k-1$ with boundary formed by identifying the sides of a $(8k-4)$-gon consisting of $2k^2-2k+1$ squares of side $(4k-3)d$ where d is a positive integer, without corner cells as in Figure 12.1 can not be tiled with $1 \times (4k-3)$ polyomino.*

Proof. From Figure 12.1 it is clear that the surface is orientable. Label the cells in the grid in standard way. Denote the cell by $a_{i,j}$ in standard way assuming that the bottom left corner cell is $a_{1,1}$. As with other I-minoes it is straightforward to get

$$\bar{a}_{i,j} = \bar{a}_{i,j+4k-3} = \bar{a}_{i+4k-3,j}.$$

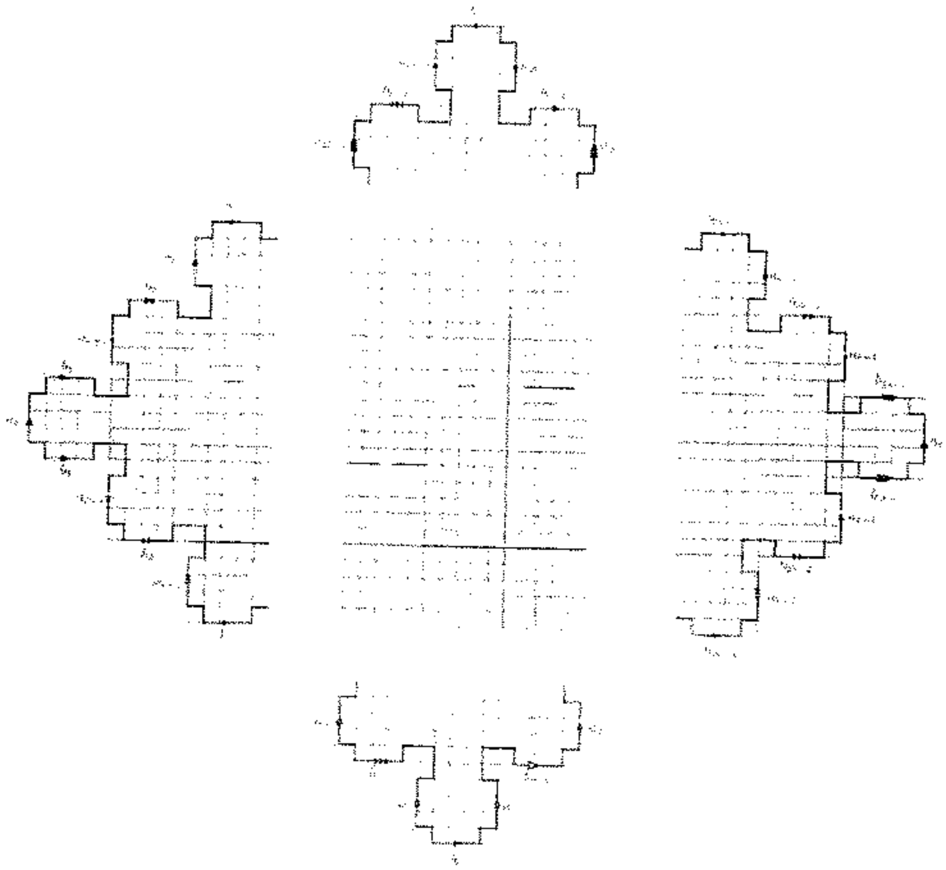

Figure 12.1: Square grid on an orientable genus $2k-1$ surface with boundary

Using this equivalences we find that there are $(4k-3)^2$ types of the cells $\bar{a}_{i,j}$, $1 \leq i,j \leq 4k-3$ in the homology group of tiling. We see that there are $8k-6$ relations

$$\sum_{j=1}^{4k-3} \bar{a}_{i,j} = 0 \quad \text{for } i = 1, \ldots, 4k-3 \quad \text{and}$$

$$\sum_{i=1}^{4k-3} \bar{a}_{i,j} = 0 \quad \text{for } j = 1, \ldots, 4k-3$$

assigned to a placement of $1\times(4k-3)$ polyomino on the board (including placements across gluing sides). Therefore, our homology group of tiling is isomorphic to

$$G\langle a_{i,j} | 1 \leq i,j \leq 4k-4\rangle \cong \mathbb{Z}^{16(k-1)^2}.$$

Element Θ assigned to the grid is

$$\Theta = -(2k-1)\sum_{i=2}^{4k-4}\sum_{j=2}^{4k-4} a_{i,j}.$$

This is a non-trivial element in the homology group of tiling and the claim is therefore proved. □

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Studij Matematike i informatike je upisao 2005. godine na Pedagoškom fakultetu Univerziteta u Bihaću. Studij završava 2009. godine s prosječnom ocjenom 8,36.

U 2010. godini se upisuje na magistarski studij na Filozofskom fakultetu Univerziteta u Zenici, smjer Matematika i informatika. Magistarski studij završava 2013. godine s prosječnom ocjenom 8,86 i stiče akademsko zvanje magistar matematike i informatike. Magistarski rad pod nazivom „Kriptosistemi s javnim ključem u funkciji rješavanja problema autentifikacije i nepobitnosti" je odbranio pod mentorstvom profesora dr. sc. Bernadina Ibrahimpašića.

Od decembra 2015. godine je student doktorskih studija Matematike na Prirodno-matematičkom fakultetu Univerziteta Crne Gore, gdje je počeo saradnju sa dr. sc. Đorđem Baralićem.

Prvo radno iskustvo stekao je u „II srednjoj školi" u Cazinu gdje je radio kao profesor matematike. Od juna 2010. godine zaposlen je na Pedagoškom fakultetu Univerziteta u Bihaću, gdje je izabran u zvanje asistenta, a kasnije višeg asistenta, na oblast Algebra i metodika nastave matematike.

Koautor je univerzitetskog udžbenika. Ima nekoliko objavljenih naučnih/stručnih radova. Učestvovao je na konferencijama iz oblasti kombinatorike, algebre i metodike nastave matematike. Istraživanja rađena u okviru doktorske disertacije predstavljao je u Lyonu (Francuska), Zagrebu (Hrvatska), Beogradu (Srbija), Podgorici (Crna Gora), Berlinu i Heidelbergu (Njemačka).

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Број: 08-414
Датум, 05.03.2010 г.

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Date,

Na osnovu člana 75 stav 2 Zakona o visokom obrazovanju (Sl.list RCG br. 60/03) i člana 18 Statuta Univerziteta Crne Gore, Senat Univerziteta Crne Gore, na sjednici održanoj 25.03.2010. godine, donio je

ODLUKU
O IZBORU U ZVANJE

Dr SVJETLANA TERZIĆ bira se u akademsko zvanje **redovni profesor** Univerziteta Crne Gore za predmete: Uvod u geometriju i Algebarska topologija na osnovnom studijskom programu Matematika i Uvog u diferencijalnu geometriju na osnovnom studijskom programu Matematika i računarske nauke na **Prirodno-matematičkom fakultetu**.

REKTOR
Prof.dr Predrag Miranović

Biografija: Svjetlana Terzić

Datum i mjesto rodjenja: 17.09.1970, Podgorica, Crna Gora;

Državljanstvo: Crna Gora;

Pol: ženski

Oblasti istraživanja: Algebarska topologija, Diferencijalna geometrija

Akademski obrazovanje:

- Diplomirani matematičar – Univerzitet Crne Gore, 09. 1993., srednja ocjena 9,96 od ukupno 10
- Magistar matematike – Univerzitet u Beogradu, 06. 1996.,srednja ocjena 10 od ukupno 10
  Magistarska teza "Karakteristične klase hiperkompleksnih vektorskih raslojenja", mentor Prof. Novica Blažić
- Doktor matematike – Moskovski državni univerzitet "M. V. Lomonosov", 02. 1996 – 07. 1998, srednja ocjena 5 od ukupno 5
  Doktorska disertacija "Realne kohomologije i karakteristične klase uopštenih simetričnih prostora ", menotor Prof. Yuri P. Solovyov, 06. 1998.
- Postdoktorska pozicija, 08. 2000- 08. 2002, Ludwig Maximillians University, Minhen, Njemačka

Akademska zvanja:

- 1993 -2000 – saradnik u nastavi, Univerzitet Crne Gore
- 2000-2005 - Docent, Univerzitet Crne Gore
- 2005-2010 – Vanredni profesor, Univerzitet Crne Gore
- 2010- Redovni profesor, Univerzitet Crne Gore
- 2011 –Vanredni član Crnogorske akademije nauka i umjetnosti
- 2018 – Redovni član Crnogorske akademije nauka i umjetnosti

Odabrana predavanja po pozivu:

1. Contemporary Geometry and Related Topics, Belgrade, Serbia and Montenegro, May 2002;
2. Kolmogorov and Contemporary Mathematics, Moscow, Russia, Jun 2003;
3. Mathematical, Theoretical and Phenomenological Challenges Beyond Standard Models, Vrnjačka Banja, Serbia and Montenegro, September 2003;
4. Algebraic models for topological spaces and fibrations, Tbilisi, Georgia, September 2004
5. XI congress of mathematicians of Serbia and Montenegro, Petrovac, Serbia and Montenegro , September 2004, plenary talk;
6. Topology, analysis and applications to mathematical physics, Moscow, Russia, February 2005;
7. Contemporary Geometry and Related Topics, Belgrade, Serbia and Montenegro, July 2005;
8. Toric Topology, Osaka, Japan, May, 2006;
9. Differential Equations and Topology, in commemoration of the 100th anniversary of L.S. Pontryagin, Moscow, Russia, Jun 2008;
10. New horizons in toric topology, Manchster, UK. July 2008;
11. Geometry, Dynamics, Integrable systems, Belgrade, Serbia, September 2008;
12. Multidisciplinarnost i jedinstvo savremene nauke, University of East Sarajevo, Pale, May 2009;
13. Geometry, topology and algebra, dedicated to 120th anniversary of Boris Delone, Steklov Mathematical Institute, Russian Academy of Science, Moscow, Russia, August , 2010;
14. Geometry, Dynamics, Integrable Systems, Belgrade, Serbia, September, 2010;
15. Toric topology and automorphic functions, Khabarovsk, Far eastern branch of Russian academy of science, September, 2011
16. International topological conference "Alexandroff readings, Moscow state university "M. V. Lomonosov", May 2012, plenary talk
17. The second mathematical conference of the Republic of Srpska, Trebinje, Jun, 2012, plenary talk
18. Geometric structure on manifolds and their applications, Castle Rauischholzhausen, Marburg, July, 2012.
19. XVII geometrical seminar, Zlatibor, Serbia, August, 2012.

21. International conference "Algebraic topology and Abelian function" in honor of Victor Buchstaber on occasion of his 70^{th} birthday, Moscow, June, 2013.
22. Geometry and analysis of metric structures, Sobolov institute of mathematics, Russian Academy of Sciences, Novosibirsk, December, 2013.
23. Topology of torus actions and its applications to geometry, Satelite conference of ICM, Daejeon, Korea, August, 2014.
24. International conference "Torus actions in geometry, topology and applications, Skolkovo, Moscow, February, 2015.
25. . The fifth mathematical conference of the Republic of Srpska, Trebinje, Jun 2015.
26. International Chinese-Russian conference " Torus actions: topology, geometry and number theory, Beijing, China, October, 2015.
27. Aspects of Homotopy Theory, Southampton, UK, December 2015.
28. XIX Geometrical Seminar, Zlatibor, Serbia, September 2016
29. Mini conference celebrating of 30 years of CGTA seminar, Belgrade, Serbia, September 2016, plenary talk
30. The Princeton-Rider Workshop on the Homotopy Theory of Polyhedral products, Princeton and Rider University, Princeton, USA, May-June, 2017.
31. Symposium on mathematics and it applications, Belgrade, Serbia, November 2017.
32. International conference "Algebraic topology, Combinatorics and Mathematical Physics" in honor of Victor Buchstaber on occasion of his 75^{th} birthday, Moscow, May, 2018.
33. International conference "Modern algebra and Analysis and their Applications, Academy of Sciences and Arts of Bosnia and Herzegovina, Sarajevo, September, 2018.
34. Susret matematičara Srbije i Crne Gore, Budva, Oktobar, 2019.
35. Toric topology 2019 in Okayama, Okayamo, Japan, Novembar, 2019.
36. Deseti simpozijum Matematika i primene, Beograd, Decembar, 2019
37. Workshop on Torus actions in Topology, Fields Institute, Toronto, Kanada, May, 2020, via zoom
38. Workshop on toric topology, geometry and related subjects, Moscow, November , 2020, via zoom

Predavanja na seminarima:

- September 2002., Erwin-Schroedinger institute, Vienna, Austria, talk in the framework of the program Aspects of foliation theory;
- April 2005, SANU, Belgrade, talk at the na Mathematical Colloquium SANU;
- Jun 2006, Osaka City University, Japan, talk at the Topology seminar;
- Jul 2006, University of Aberdeen, UK, talk at the Topology seminar
- January 2007, University of Oxford, UK, talk at the Topology seminar, mini course for phd topology students on the rational minimal model theory;
- February 2007, University of Manchester, UK, talk at the Topology seminar;
- November 2007, Mathematical Institute SANU, Belgrade, talk at the Geometry seminar;
- April 2009, MFO (Oberwolfach), Germany , talk at the "Workshop on homotopy theory of function spaces and related topics";
- September 2009, Faculty of Mechanics and Mathematics, MSU "M. V. Lomonosov", Moscow, Russia, talk at the seminar for Geometry, topology and mathematical physics, chaired by V. M. Buchstaber and S. P. Novikova , talk at the Chair seminar of A. T. Fomenko;
- Mart 2010, Laboratori J. A. Dieudonne, Universite de Nica Sophia Antipolis, France, talk at the seminar for Algebra, topology and geometry;
- Jun 2010, International School for Advanced Studies SISSA, Trieste, Italy, talk at the seminar for Geometry and Physics chaired by B. A. Dubrovin;
- December 2011, SANU, talk at the seminar Mathematical methods of mechanics.
- September 2013, University of Southampton, talk at Topology seminar
- December 2013, SANU, talk at the seminar Mathematical methods of mechanics
- October 2016, Faculty of Mechanics and Mathematics, MSU "M. V. Lomonosov", Moscow, Russia, talk at the seminar for Geometry, topology and mathematical physics, chaired by V. M. Buchstaber and S. P. Novikov
- May 2017, University of Southampton, UK; talk at Topology seminar
- December 2018, University of Southampton, UK, talk at Topology seminar

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- November 2019, talk at One day topology seminar in Osaka, Osaka, Japan
- Oktobar 2020, University of Southampton, talk at Topology seminar, via zoom
- Novembar 2020, Princenton University, talk at International Polyhedral Product seminar, via zoom

Odabrane nagrade i grantovi:

1. Nagrada 19. decembar za najboljeg studenta u generaciji 1991.
2. Plaketa Univerziteta Crne Gore za najboljeg diplomiranog studenta generacije, 1993.
3. Nagrada Crnogorske akademije nauka i umjetnosti za naučna dostignuća, 2003.
4. Grant Evropskog udruženja matematičara za učešće na IV Evropskom kongresu matematičara, Štokholm, Švedska, 2004
5. WUS-Austria 2-nedjeljna posjeta Jelene Grbić Podgorici, Crna Gora, April, 2007.
6. Oxford Colleges hospitality scheme, 1-mjesečna posjeta Univerzitetu u Oxford-u , Januar, 2007.

7. Grant Evropskog udruženja matematičara za učešće na V Evropskom kongresu matematičara, Amsterdam, Holandija, 2008.
8. Grant of the Medjunarodne matematičke unije za učešće na Svjetskom kongresu matematičara, Hyderabad, Indija, 2010.
9. Bilateralni projekat sa Univerzitetom u Ljubljani, Slovenija, 2012-2013.
10. Glavni istraživač na projekstu Ministarstva nauke Crne Gore, 2012-2015.
11. Glavni istraživač na medjunarodnom projektu instituta SISSA, Trst, 2008-2010.
12. Spoljni istraživač na projektu 174020 Ministarstva nauke Srbije, 2011-2015.
13. Istraživački grant London Mathematical Society sa Jelenom Grbić, 2-nedjeljna posjeta Univerzitetu u Southampton-u, 2013.
14. Grant za istraživanje u parovima sa Jelenom Grbić, 3-nedjeljna posjeta Matematičkom institute Oberwolfach, 2014.
15. Grant Medjunarodne matematičke unije za učešće na Svjetskom matematičkom kongresu Rio de Janeiro, Brazil, 2018.

Neke naučno istraživačke posjete:

Mehaniko-matematički fakultet, Moskovski državni univerzitet,Matematički institut Steklova, Ruska akademija nauka – 1999, 2003, 2005, 2006, 2008, 2009, 2012, 2013, 2015, 2016, 2018; SISSA Trst 2010; Mathematički Fakultet, Ljubljana, 2012, 2013; Matematički fakultet, Univerzitet u Southampton-u, 2013, 2015, 2017; Univerzitet u Aberdeen-u, 2006; Univerzitet u Mančester-u, 2007 . Fildsov Institu za matematiku, Toronto, Kanada, 2020.

Nastava i mentorstvo:

Predavala kurseve na različitim nivoima studija na Prirodno-matematičkom fakultetu Univerziteta Crne Gore: Uvod u geometriju, Uvod u diferencijanu geoemtriju, Algebarska topologija, Diferencijalna geomatrija na mnogostrukostima, Geometrija, Napredna algebra.

Mentor za preko 20 specijalističkih radova i 4 magistraske teze, komentor doktorske disertacije na Matematičkom fakultetu, Univerzitet Nica Sophia Antipolis, član komisija za odbranu doktorskih disertacija na Univerzitetu u Beogradu , Univerzitetu u Istočnom Sarajevu, Univerzitetu u Southmpton-u, Univerzitetu Crne Gore.

Ostalo:

- Urednik:
  1. Sarajevo Journal of Mathematics, izdaje Akademija nauka i umjetnosti Bosne i Hercegovine
  2. Matematički Vesnik, izdaje Društvo matematičara Srbije

- Recenzent za časopise : Publication de l'Institute Mathematique, Contemporary Mathematics, Proceedings of the Steklov Institute of Mathematics, Annali di Matematica Pura ed Applicata, Mathematica Slovaka, Mathematische Zeitschrift, Sbornik:Mathematics, Algebraic and Geometric Topology, Homology, Homotopy and Applications, Morrocan Journal of Pure and Applied Analysis

- Prodekan za medjunarodnu sardanju na Prirodno-matematičkom fakultetu Univerziteta Crne Gore, 2004 – 2007.

Publication list for Svjetlana Terzić

1. Svjetlana Terzić, *Real cohomology and Pontryagin characteristic classes of generalised symmetric spaces*, (Russian) Vsesojuzni Institut Nauchnoj i Tehnicheskoj Informacii, VINITI, V-1034, Moscow, 1998, 1-94.
2. Svjetlana Terzić, *Generalised symmetric spaces and their topology*, (Russian) Mathematica Montisnigri 11 (1999), 139-150.
3. Svjetlana Terzić, *Characteristic classes of hypercomplex vector bundles*, Montenegrin Academy of Sciences and Arts, Proceeding of the Section of Natural Sciences, 13 (2000)
4. Svjetlana Terzić, *Cohomology with real coefficients of generalized symmetric spaces*, (Russian) Fundamentalnaya i Prikladnaya Matematika, Vol. 7, (2001), no. 1, 131-157.
5. Svjetlana Terzić, *Pontryagin classes of generalized symmetric spaces*, (Russian) Matematicheskie Zametki, Vol. 69, (2001), no.4, 613-621; English transl. in Mathematical Notes, Vol. 69, (2001), no. 4, 559-566.
6. D. Kotschick and S. Terzić, *On formality of generalised symmetric spaces*, Mathematical Proceedings of Cambridge Philosophical Society, 134 (2003), 491-505.
7. S. Terzić, *Rational homotopy groups of generalised symmetric spaces*, Mathematische Zeitschrift, 243 (2003), 491-523.
8. S. Terzić, *On rational topology of four manifolds*, Proceeding of the Workshop Contemporary Geometry and Related Topics, World Scientific 2004, 375-389.
9. Svjetlana Terzić, *Rational topology of gauge groups and of spaces of connections*, Compositio Mathematicae, 141 (2005), no.1, 262-270.
10. Svjetlana Terzić, *On geometric formality*, Proceedings of the Workshop devoted to 25th anniversary of the Faculty of Natural Sciences and Mathematics, University of Montenegro, Contemporary mathematics, physics and biology, (2006), 208-215.
11. Victor M. Buchstaber and Svjetlana Terzić, *Equivariant complex structures on homogeneous spaces and their cobordism classes*, Advances in the Mathematical Sciences, Geometry, topology and mathematical physics, Translations 2, 224 (2008), 27 – 57, American Mathematical Society
12. D. Kotschick and S. Terzić, *Chern numbers and the geometry of partial flag manifolds*, Commentarii Mathematici Helvetici, 84 (2009), no.3, 587 – 616.
13. Jelena Grbić and Svjetlana Terzić, *The integral Pontryagin homology of the based loop space on a flag manifold*, Osaka Journal of Mathematics 47 (2010), no 2, 439 – 460.
14. Svjetlana Terzić, *Integral loop homology of complete flag manifolds* (joint with Jelena Grbić), Oberwolfach reports 19/2009, Homotopy Theory of Function Spaces and Related Topics, European Mathematical Society Publishing House, 1038 – 1040.

15. D. Kotschick and S. Terzić, *Geometric formality of homogeneous spaces and of biquotients*, Pacific Journal of Mathematics, 249 (2011), no 1, 157 – 176.

16. Svjetlana Terzić, *On real cohomology generators of compact homogeneous spaces*, Sarajevo Journal of Mathematics, Vol. 7 (20) (2011), No. 2, 277 – 287 .

17. Svjetlana Terzić, *Toric genera on homogeneous spaces and related problems*, Proceedings of the international conference "Toric topology and automorphic functions", Far-Eastern Branch of the Russian Academy of Sciences, Pacific National University, 2011, 97 – 104.

18. Jelena Grbić and Svjetlana Terzić, *The integral homology ring of the based loop space on some generalised symmetric spaces*, Moscow Mathematical Journal, Volume 2012, Issue 4, Oct. – Dec. 2012, pp 771-786.

19. Victor M. Buchstaber and Svjetlana Terzić, *Toric genera of homogeneous spaces and their fibrations*, International Mathematics Research Notices, Vol. 2013, 1324-1403.

20. Svjetlana Terzić, *On cohomology ring of partial flag manifolds*, Proceedings of the Second Mathematical Conference of the Republic of Srpska, 2013, 11 - 17. (ISBN 978 – 99938 – 47 – 52 – 6)

21. Svjetlana Terzić, *Rational minimal model theory on compact homogeneous spaces*, Scripta Scientiarum Naturalium, Proceedings of the Faculty of Natural Sciences and Mathematics, University of Montenegro, Vol. 3. 2013, 3 – 17. (ISSN 1880 – 8356)

22. Victor M. Buchstaber and Svjetlana Terzić, *"(2n,k)-manifolds and applications"*, Report No 27/2014, Mathematisches Forschung Institut (25–31 May 2014), Oberwolfach, Germany, 2014, 11–14.

23. Victor M. Buchstaber and Svjetlana Terzić, *Topology and Geometry of the Canonical Action of T^4 on the complex Grassmannian $G_{4,2}$ and the complex projective space CP^5*, Moscow Mathematical Journal, Vol. 16, Issue 2 (2016), 237-273.

24. Svjetlana Terzić, *Geometric formality of rationally elliptic manifolds in small dimensions*, Glasnik of the Section of Natural Sciences, Montenegrin Academy of Sciences and Arts, 20 (2014), 131-145.

25. Svjetlana Terzić, *The rational homology ring of the based loop space of the gauge group and the spaces of connections on a four manifold*, Fundamentalnaya i Prikladnaya matematika (in Russian), Vol. 21, No.6, (2016) 206-216.

26. Svjetlana Terzić, *On geometric formality of rationally elliptic manifolds in dimensions 6 and 7*, Publications de l'Institute Mathematique (Belgrade), Issue 103 (117), (2018), 211-222.

27. Svjetlana Terzić, *Rational Pontrjagin homology ring of the based loop space on some homogeneous spaces*, Sarajevo Journal of Mathematics, Vol. 14, No. 2, (2018), 275-285.

28. Victor M. Buchstaber and Svjetlana Terzić, *The foundations of (2n, k)-manifolds*, Sbornik Mathematics, 210:4 (2019), 41 – 86.

29. Victor M. Buchstaber and Svjetlana Terzić, *Toric topology of the complex Grassmann manifolds*, Moscow Mathematical Journal, 19:3 (2019), 397-463.

30. Dragana Borović and Svjetlana Terzić, *On rational Pontryagin homology ring of the based loop space on a four-manifold*, Matematički Vesnik, Vol 71, Iss 1-2, (2019), 90-103.

Република Србија
МИНИСТАРСТВО ПРОСВЕТЕ,
НАУКЕ И ТЕХНОЛОШКОГ РАЗВОЈА
Комисија за стицање научних звања

Број: 660-01-00001/1255
10.06.2020. године
Б е о г р а д

На основу члана 22. став 2. члана 70. став 5. Закона о научноистраживачкој делатности ("Службени гласник Републике Србије", број 110/05, 50/06 – исправка, 18/10 и 112/15), члана 3. ст. 1. и 3. и члана 40. Правилника о поступку, начину вредновања и квантитативном исказивању научноистраживачких резултата истраживача ("Службени гласник Републике Србије", број 24/16, 21/17 и 38/17) и захтева који је поднео

Математички институт САНУ у Београду

Комисија за стицање научних звања на седници одржаној 10.06.2020. године, донела је

ОДЛУКУ
О СТИЦАЊУ НАУЧНОГ ЗВАЊА

Др Ђорђе Баралић

стиче научно звање
Виши научни сарадник

у области природно-математичких наука - математика

О Б Р А З Л О Ж Е Њ Е

Математички институт САНУ у Београду

утврдио је предлог број 304/5 од 02.09.2019. године на седници Научног већа Института и поднео захтев Комисији за стицање научних звања број 304/4 од 02.09.2019. године за доношење одлуке о испуњености услова за стицање научног звања ***Виши научни сарадник.***

Комисија за стицање научних звања је по претходно прибављеном позитивном мишљењу Матичног научног одбора за математику, компјутерске науке и механику на седници одржаној 10.06.2020. године разматрала захтев и утврдила да именовани испуњава услове из члана 70. став 5. Закона о научноистраживачкој делатности ("Службени гласник Републике Србије", број 110/05, 50/06 – исправка,18/10 и 112/15), члана 3. ст. 1. и 3. и члана 40. Правилника о поступку, начину вредновања и квантитативном исказивању научноистраживачких резултата истраживача ("Службени гласник Републике Србије", број 24/16, 21/17 и 38/17) за стицање научног звања ***Виши научни сарадник,*** па је одлучила као у изреци ове одлуке.

Доношењем ове одлуке именовани стиче сва права која му на основу ње по закону припадају.

Одлуку доставити подносиоцу захтева, именованом и архиви Министарства просвете, науке и технолошког развоја у Београду.

ПРЕДСЕДНИК КОМИСИЈЕ

Др Ђурђица Јовановић,
научни саветник

МИНИСТАР

Младен Шарчевић

РЕПУБЛИКА СРБИЈА МИНИСТАРСТВО ПРОСВЕТЕ НАУКЕ И ТЕХНОЛОШКОГ РАЗВОЈА БЕОГРАД

Ђорђе Баралић- Биографија

Ђорђе Баралић је рођен 29.09.1986. у Крагујевцу, где је завршио основну школу и Прву крагујевачку гимназију. Дипломирао је 2008. на теоријској математици на Природно-математичком факултет Универзитета у Крагујевцу. Докторску дисертацију из области алгебарске топологије одбранио је 2013. на Математичком факултету у Београду. Од 2008. запослен је на Математичком институту САНУ где је 2014. изабран у звање научног сарадника, а 2020. у звање вишег научног сарадника. Од 2021. је на месту помоћника директора за међународну сарадњу, рад са талентима за математику, рачунарство и механику, и промоцију математичких и рачунарских наука.

Области његовог научног интересовања су торусна топологија, комбинаторика, геометрија и примене. Аутор је 18 научних радова публикованих у референтним научним часовима и једне збирке задатака. Ментор је две докторске дисертације. Током досадашње каријере учествовао је одржао предавања на преко 40 конгреса, конференција и летњих школа широм света. Био је позвани млади истраживач на Институту Енио ди Ђорђи у Пизи, Институту за математичке науке Националног универзитета у Сингапуру и Вијетнамског института за напредне студије у математици у Ханоју, а боравио је и на Филдсовом математичком институту у Торонту, Математичком институту у Оберволфаху, Институту за математичка истраживања у Стразбуру, Институту Митаг-Лефлер у Штокхолму и Стекловом математичком институту у Москви.

Учествовао је на реализацији два Хоризонт 2020 пројекта и руководилац је два билатерална пројекта са Словенијом и Турском. Био је један од организатора радионица на 14. Српском математичком конгресу, Математичке конференције Републике Српске и Првог сусрета математичара Србије и Црне Горе. Одржао је предавања по позиву у Словенији, Црној Гори, Босни и Херцеговини, Хрватској, Македонији, Аргентини, Вијетнаму, Француској, Русији, Немачкој и Уједињеним Арапским Емиратима.

Два пута је учестовао на Хајделбуршком форуму лауреата. Активно учествује у популаризацији математике, а од 2017. у сарадњи са Центром за промоцију науке Србије организује манифестацију Мај месец математике. Говори енглески и шпански језик.

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ODLUKU
O IZBORU U ZVANJE

Dr ŽANA KOVIJANIĆ-VUKIĆEVIĆ bira se u akademsko zvanje **redovni profesor** Univerziteta Crne Gore za predmete: Diskretna matematika I, Diskretna matematika II i Uvod u kombinatoriku, na **Prirodno-matematičkom fakultetu.**

REKTOR

Prof.dr Predrag Miranović

БИОГРАФИЈА

Жана Ковијанић Вукићевић је рођена 16. јуна 1967. године у Подгорици, гдје је завршила основну и средњу школу. Учесник је републичких и савезних (СФРЈ) такмичењима из математике, физике и историје на којима је освајала признања и медаље.

Природно-математички факултет Универзитета Црне Горе уписала је 1985. и дипломирала октобра 1989. са просјечном оцјеном 9.93. Добитник је Децембарске награде и Плакете Универзитета Црне Горе за школску 1988/89. годину.

Послиједипломске студије уписала је у децембру 1989. на Математичком факултету Универзитета у Београду, гдје је све испите положила са оцјеном 10 и априла 1994. године одбранила магистарски рад „ε-мреже и још неки проблеми дискретне геометрије". Период септембар 1996 – јун 1998. провела је на усавршавању на Московском државном универзитету „М. В. Ломоносов " на Механичко-математичком факултету, катедра за математичку теорију интелектуалних система (МаТИС). Докторску дисертацију „Комбинаторно-вјероватносни метод у проблемима пребројавања к-значне логике" одбранила је у фебруару 2000. године на Математичком факултету Универзитета у Београду, под менторством проф. др Радета Живаљевића. Након повратка из Москве задржала је контакт са сарадницима катедре МаТИС и више пута боравила на овој катедри.

Била је члан више националних научно-истарживачких пројеката и вођа три билатерална пројекта. Више година активно је учествовала у организацији математичких такмичења, и била вођа екипе Црне Горе на Међународним математичким олимпијадама.

Жана Ковијанић Вукићевић је запослена на Природно-математичком факултету од 9. октобра 1989. У звање редовног професора Универзитета Црне Горе изабрана је октобра 2010. године.

Библиографија (изабрани радови – до 10)

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Dr Vladimir Božović bira se u akademsko zvanje redovni profesor Univerziteta Crne Gore za **oblast Matematika,** na Prirodno-matematičkom fakultetu Univerziteta Crne Gore, na neodređeno vrijeme.

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Research Interests

Group Theory Factorization of groups with application to cryptography and coding theory.
Biometrics Design and analysis of safe and secure biometric systems.
Combinatorics Enumerative combinatorics, graph theory.

Education

Florida Atlantic University, Boca Raton, Florida USA

Ph.D., Mathematics, December 2008.

- Dissertation Topic: "Algebraic and Combinatorial Aspects of Group Factorizations."
- Advisor: Spyros S. Magliveras

M.S., Mathematics, May 2006.

University of Belgrade, Belgrade, Serbia.

M.Sc., Mathematics, March, 2003.

- Dissertation Topic: "The Isomorphism Problem For Group Rings".
- Advisor: Slobodan Vujošević.

University of Montenegro, Podgorica, Montenegro.

B.A, Mathematics, September, 1999.

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Center for Cryptology and Information Security (CCIS)

Research assistant — **Summer 2005**
Algebraic properties of the crypto system PGM and MST1.
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Research assistant — **Summer 2008**
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Florida Atlantic University, Boca Raton, Florida, USA

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- Introductory Number Theory, Fall 2006.
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Teaching assistant
Shared Responsibility for lectures, exams, homework assignments, and grades.

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Infoteh, Jahorina, Republic of Srpska, Bosnia and Hercegovina, "3D Rekonstrukcija koristeći slike sa interneta i algoritam postepenog rasta regiona", 15-16 mart 2011.

Kongres matematičara i fizičara Crne Gore, KMFCG 2010, Petrovac, Crna Gora, "Osvrt na kriptografiju", 7-10 oktobar 2010.

Kongres matematičara i fizičara Crne Gore, KMFCG 2010, Petrovac, Crna Gora, "Faktorizacija konačnih grupa", 7-10 oktobar 2010.

10th International Conference on Computational and Mathematical Methods in Science and Engineering CMMSE 2010, Alicante, Spain, "Multi-authority attribute based encryption with honest-but-curious central authority", June 26-29, 2010.

2008 Southern Regional Algebra Conference, University of Colorado, Colorado Springs, Colorado, USA, "Free mappings and factorization of groups", September 26-28, 2008.

Thirty-ninth Southeastern International Conference on Combinatorics, Graph Theory, and Computing, Boca Raton, Florida, USA, "Bipartite graphs with no isolated vertices and k-tuples of discrete intervals", March 3-7, 2008.

International Conference on Security and Cryptography (SECRYPT 2007), Barcelona, Spain, "Practical Secure Biometrics Using Set Intersection as a Similarity Measure", July 28-31, 2007.

International Conference on Risks and Security of Internet and Systems (CRiSIS 2007), Marrakech, Morocco, "Issues and Challenges in Storing Biometric Templates Securely", July 2-5, 2007.

Integers conference, University of West Georgia, Carrollton, Georgia, USA, "The distribution of the Size of the Intersection of a k-Tuple of Intervals", October 27-30, 2005.

Thirty-sixth Southeastern International Conference on Combinatorics, Graph Theory, and Computing, Boca Raton, Florida, USA, "The distribution of the Size of the Intersection of a k-Tuple of Intervals", March 7-11, 2005.

SEMINAR PRESENTATIONS

Department of Mathematics, Florida Atlantic University, Algebra and Crypto seminar, "Rank 3 permutation groups and block designs", September 19, 2006.

Mathematical Institute of Serbian Academy od Sciences and Arts, Belgrade, Serbia, "The problem of isomorphism of group rings", November 22, 2002.

THESES AND DISSERTATIONS

Vladimir Božović, "Algebraic and Combinatorial Aspects of Group Factorizations", Ph.D. dissertation, Department of Mathematical Sciences, Florida Atlantic University, December 2008.

Vladimir Božović, "The Isomorphism Problem for Group Rings", M.Sc. thesis, Faculty of Mathematics, University of Belgrade, March 2003.

COMPUTER SKILLS

- Programmable Environments: MS Visual Studio, Dreamweaver, Maple, APL, Magma, GAP, LaTeX
- Languages: C/C++, Pascal, HTML, PHP, XHTML and CSS

LANGUAGES

Serbian (native language), **English** (fluent).

Република Србија
МИНИСТАРСТВО ЗА НАУКУ И ТЕХНОЛОГИЈУ
Комисија за стицање научних звања
Број: 06-00-6/429
01.11.1995. године.
Београд

На основу члана 40. става 4. Закона о научноистраживачкој делатности ("Службени гласник Републике Србије", број 52/93), и захтева који је поднео
Математички институт САНУ у Београду
на седници Комисије за стицање научних звања Министарства за науку и технологију одржаној 01.11.1995. године, донета је

ОДЛУКА
О СТИЦАЊУ НАУЧНОГ ЗВАЊА

Др Раде Живаљевић

стиче научно звање

Научни саветник

ОБРАЗЛОЖЕЊЕ

Математички институт САНУ у Београду
утврдио је предлог одлуке број 256/2 од 28.09.1995. године на седници научног већа Института и поднео захтев Министарству број 121/5 од 29.09.1995. године за оцену о испуњености услова за стицање научног звања Научни саветник.

Комисија за стицање научних звања Министарства за науку и технологију на седници одржаној 01.11.1995. године разматрала је захтев и утврдила да именовани испуњава услове из члана 34. Закона о научно-истраживачкој делатности за стицање научног звања ***Научни саветник*** па је одлучила као у изреци ове одлуке.

Одлука је коначна.

Одлуку доставити подносиоцу, именованом и архиви Министарства за науку и технологију у Београду.

ПРЕДСЕДНИК КОМИСИЈЕ
Академик проф. др Иван Спужић

Rade T. Živaljević
Mathematical Institute SASA, Belgrade
Scientific Biography (abridged version)

1 Education

Rade Živaljević was born on October 12, 1954 in Sarajevo. Master thesis (M.Sci) and Ph.D. degree thesis were defended at Belgrade University in 1979, respectively 1983. The second Ph.D. degree thesis was defended at the University of Wisconsin-Madison in 1985. He has been a member of the Mathematical Institute of the Serbian Academy of Sciences and Arts (SASA) since 1977, where he was appointed as a full research professor in 1995.

2 Scientific Activities

2.1 Selected visits

Visiting associate professor at the University of Illinois at Urbana-Champaign in 1994; Mittag-Leffler institute (Stockholm, Year of Combinatorics, 1991); Konrad Zuse Zentrum für Informationstechnik (ZIB), Berlin, 1994); Mathematical Institute Bern (1999). Shorter periods at Institut des Heutes Études (Paris), Mathematical Sciences Research Institute, Berkeley (2006, 1998, 1991), Moscow State University (MGU), Mathematics Forschungs Institut (Oberwolfach), Givat-Ram (Jerusalem), Technion (Haifa), KTH Stockholm, SISSA (Trieste), etc. The most recent visits include Moscow State University (April 2016), "Reseach in Pairs" program (Mathematisches Forschungsinstitut Oberwolfach, May 2016), Brown University (Institute for Computational and Experimental Research in Mathematics (ICERM), program "Topology in Motion", Fall 2016), Saint Petersburg State University (June, 2018), Research in pairs, Centre International de Rencontres Mathématiques (CIRM, Marseille), fall 2018, "Reseach in Pairs" program (Mathematisches Forschungsinstitut Oberwolfach, 2019).

2.2 Conferences

Selected lectures and addresses include the following:
Classical and Contemporary Geometry, Moscow 1-4. 11. 2021, Geometric Topology and Hypergraphs MOCCA 2021 mini symposiym, Moscow, MIPT June 1, 2021. "Toric Topology 2019 in Okayama", Okayama (Japan) Fall 2019 (invited lecture), "Algebraic Topology, Combinatorics and Mathematical Physics" in honor of Victor Buchstaber on the occasion of his 75th birthday. Steklov Mathematical Institute of the Russian Academy of Sciences and Skolkovo Institute of Science and Technology (SkolTech), Moscow, 24—30 May 2018 (plenary lecture). Steklov Mathematical Institute RAS (St. Petersburg, 6.6.2018). Chebishev Laboratory of St. Petersburg State University (June 2018), Applied Topology in Bedlewo 2017 25 June 2017 - 1 July 2017, Bedlewo (Poland). Princeton Algebraic Topology Seminar (October 2016), M.I.T. Topology Seminar (October, 2016), 19 Geometric Seminar (Zlatibor,

August 2016), Seminar Discrete and Computational Geometry (Moscow, MIPT, April 2016), The Fifth German-Russian Week of the Young Researcher on Discrete Geometry, MIPT Moscow, 6-11 September 2015; Summer school on computational topology (Ljubljana 2015), Geometric and Algebraic Combinatorics, Oberwolfach 2015; Applied Algebraic Topology, Castro Urdiales 2014, Geometry, Topology, Integrability, Moscow (Skolkovo) 2014; Applied Topology Bedlewo 2013, Algebraic Topology and Abelian Functions - Moscow, 2013; Geometry, Topology, Algebra and Number Theory, Moscow (Steklov Inst.), 2010; Gil Kalai i R. Živaljević are organizers of the conference Combinatorics and Topology, Jerusalem, June 19 - June 22, 2007; Technical University Berlin and Free University Berlin, colloquium speaker (December 2005); Algebraic and Geometric Combinatorics, Anogia, Greece (August 2005); Combinatorics Symposium in honor of Helge Tverberg, Bergen (March 2005), Trends in Topological Combinatorics, KTH Stockholm (February 2005); 18 British Topology Symposium, Manchester (September 2003), Workshop on Topological Methods in Combinatorics, KTH Stockholm, May 31 - June 2, 2006, etc.

2.3 National and international projects

R. Živaljević is (together with Vladimir Dragović) the founder of the center "Dynamical Systems, Geometry and Combinatorics'), as a research unit (center of excellence) in Mathematical Institute SASA (Belgrade) `http://www.mi.sanu.ac.yu/dsgc/dsgc.htm`. Center is or was in the past the coordinator of international cooperation with other groups with similar orientation from the following institutions. Steklov Institute in Moscow (V. Kozlov, V. Buchštaber); Mathematical Physics Sector, SISSA - ISAS, Trieste (B. A. Dubrovin); Discrete Geometry Group, TU Berlin (G. Ziegler); DIMATIA, Prague, Czech Republic (J. Matoušek), as well as the regional groups (Banja Luka, Podgorica, Zagreb, Niš, itd.).
R. Živaljević was the coordinator, together with Petar Pavešić (Univ. of Ljubljana, Slovenia), of the Serbia-Slovenia bilateral project "Applied and Computational Algebraic Topology" (2016-2017).

2.4 Awards and honors

R. Živaljević was in 1995 awarded (together with Siniša Vrećica) the City of Belgrade Award for the solution of the "Colored Tverberg Problem". This award was at the time in Serbia the highest award for scientific research and other achievements.

2.5 Students

R.Živaljević was the PhD degree thesis adviser (University of Belgrade) of Pavle Blagojević, Vladimir Grujić, Djordje Baralić, Žana Kovijanić, Manuela Muzika-Dizdarević, and a co-adviser of Duško Jojić. He was a co-adviser of Stephan Hell (Technische Universität Berlin 2006). His current Ph.D. degree studentis Marinko Timotijević.

3 Research

3.1 Research interests

Rade Živaljević is the author of more than 50 research papers and several review and expository publications. Among the research publications are papers published in highly ranked international mathematical journals which includes *Advances in Mathematics*, *Mathematische Annalen*, *J. Reine Angew. Math.*, *Trans. Amer. Math. Soc.*, *J. London Math. Soc.*, *Combinatorica*, etc. The main contributions of Rade Živaljević are in the areas of topological and geometric combinatorics, discrete and computational geometry, applied algebraic topology.

3.2 Selected results

The main contributions may be classified in three thematic circles: Homotopy colimits and Ziegler-Živaljević-formulae; "Configuration space-test map"-scheme with applications in computational topology; Homotopic and cohomological methods in topological combinatorics.

(1) Solution of the problem (possed by Victor Vassiliev, Berkeley 1997) of describing the geometric resolutions $exp_n(S^m)$ of spheres (*Advances in Applied Mathematics*, 1998).

(2) The Csorba-Živaljević universality theorem for Lovász graph complexes (Journal of Combinatorial Theory, Ser A, 2005).

(3) The problem (Branko Grünbaum, 1960) of equipartitions of measures in the 4-dimensional euclidean space (Transactions of the American Mathematical Society, 2008).

(4) Multidimensional „Splitting necklace"-theorem (Advances in Mathematics, 2008) as an extension of the one dimensional case (Noga Alon, 1987).

(5) Combinatorial techniques for the study of symmetric cohomology of algebras and a solution of a problem of Ault and Fiedorowicz-a (*European Journal of Combinatorics*, 2009, coauthor S. Vrećica).

(6) Differential and algebraic topology of "totally skew embeddings" (Transactions Amer. Math. Society 2011, coauthors S. Vrećica, B. Prvulović, G. Stojanović, and Dj. Baralić).

(7) "Center Transversal Theorem" (Bulletin London Math. Society 1990, coauthor S. Vrećica).

(8) Ziegler–Živaljevic formulae, Mathematische Annalen 1993, J. Reine Angew. Math., 1999.

(9) Work on the colored Tverberg problem and chessboard complexes, J. Combin. Theory, Ser. A (1992 and 2011), J. London Math. Soc. 1994.

(10) Proof (together with D. Jojić and S. Vrećica, *J. Algebraic Combin.*, 46 (2017)) of the conjecture of Blagojević, Frick, and Ziegler about the existence of 'balanced Tverberg partitions' (Conjecture 6.6 in, Tverberg plus constraints, Bull. London Math. Soc. 46 (2014)).

4 Publications

4.1 Five papers with the largest number of citations

[1] G. Ziegler, R. Živaljević, Homotopy types of subspace arrangements via diagrams of spaces, *Mathematische Annalen, 295:527–548, 1993.*

[2] R. Živaljević, S. Vrećica, The colored Tverberg's problem and complexes of injective functions, *J. Combin. Theory, Ser. A* 61 (2), 1992, 309–318.

[3] A. Björner, L. Lovász, S. Vrećica, and R. Živaljević, Chessboard and matching complexes, *J. London Math. Soc.* (2), 49:25–39, 1994.

[4] V. Welker, G. Ziegler, R. Živaljević, Homotopy colimits – comparison lemmas for combinatorial applications , *J. Reine Angew. Math.*, 509 (1999), 117–149.

[5] R. Živaljević, S. Vrećica An extension of the ham sandwich theorem. *Bull. London Math. Soc. vol. 22, 1990, pp. 183–186.*

4.2 Main publications after 2003

[6] P. Blagojević, V. Grujić, R. Živaljević. Symmetric products of surfaces and the cycle index. Israel J. Math. 138 (2003) 61–72.

[7] P. Mani-Levitska, S. Vrećica, R. Živaljević, *Topology and Combinatorics of Partitions of Masses by Hyperplanes*, Advances in Mathematics 207 (2006) 266–296.

[8] R. Živaljević, *Groupoids in combinatorics – applications of a theory of local symmetries*, Proceedings of the Conference "Algebraic and Geometric Combinatorics", Anogia, Greece 2005, Contemporary mathematics A.M.S, 2007; Vol. 423, 305–324.

[9] R.T. Živaljević, Equipartitions of measures in $\mathbb{R}^4$, *Trans. Amer. Math. Soc.* Volume 360, Number 1, January 2008, pp. 153–169.

[10] M. de Longueville, R.T. Živaljevic, Splitting multidimensional necklaces. *Advances in Mathematics*, 2008, DOI: 10.1016/j.aim.2008.02.003.

[11] Pavle V. M. Blagojević, Siniša T. Vrećica, Rade T. Živaljević, Computational topology of equivariant maps from spheres to complements of arrangements, *Trans. Amer. Math. Soc.* 361 (2009), 1007-1038. GS = 7, WS = ?

[12] R. Živaljević, Combinatorial Groupoids, Cubical Complexes, and the Lovász Conjecture, *Discrete & Computational Geometry*, Volume 41 , Issue 1 (January 2009), pp. 135-161.

[13] S. Vrećica, R. Živaljević, Cycle-free chessboard complexes and symmetric homology of algebras, *European Journal of Combinatorics* Volume 30 , Issue 2 (February 2009), pp. 542-554.

[14] R. Živaljević, Oriented matroids and Ky Fan's theorem, *Combinatorica*, (2010), vol. 30 br. 4, str. 471–484.

[15] S. Vrećica, R. Živaljević, Fulton-MacPherson compactification, cyclohedra, and the polygonal pegs problem, *Israel J. Math.*, (2011), vol. 184 no. 1, pp. 221–249.

M21

[16] S. Vrećica, R. Živaljević, Chessboard complexes indomitable, Journal of Combinatorial Theory Series A, (2011), vol. 118 br. 7, str. 2157–2166. **M21**

[17] Dj. Baralić , B. Prvulović , G. Stojanović , S. Vrećica, R. Živaljević, Topological obstructions to totally skew embeddings. *Trans. Amer. Math. Soc.*, (2012), Vol. 364, 2213–2226. **M21a**

[18] R. Živaljević, Rotation number of a unimodular cycle: an elementary approach, *Discrete Mathematics* (2013), vol. 313, 2253–2261.

[19] M. Muzika-Dizdarević, R. Živaljević, Symmetric polyomino tilings, tribones, ideals, and Groebner bases, *Publ. Inst. Math. (Beograd) (N.S.)* 98 (112) (2015), 1–23. **M23** (2014)

[20] R. Živaljević, Computational Topology of Equipartitions by Hyperplanes, Topological Methods in Nonlinear Analysis, (2015), vol. 45, 63–90. **M21**

[21] R. Živaljević, Illumination complexes, Delta-zonotopes, and the polyhedral curtain theorem, Computational geometry-theory and applications, (2015), 225–236. **M22**

[22] S. Vrećica, R. Živaljević. Measurable Patterns, Necklaces and Sets Indiscernible by Measure. Topological Methods in Nonlinear Analysis, (2015), vol. 45 br. 1, 39–53. **M21**

[23] M. Muzika-Dizdarević, R. Živaljević. Signed Polyomino Tilings By n-in-Line Polyominoes and Gröbner Bases, *Publ. Inst. Math. (Beograd) (N.S.)* 99 (113) (2016), 31–42. **M23** (2014)

[24] R.T. Živaljević. A glimpse into continuous combinatorics of posets, polytopes, and matroids, Fundam. Prikl. Mat., 2016, Volume 21, Issue 6, 143–164. (transl.) Journal of Mathematical Sciences (Springer), `http://www.mathnet.ru/php/journal.phtml?jrnid=fpm&option_lang=rus` **M24**

[25] D. Jojić, S.T. Vrećica, R.T. Živaljević. Multiple chessboard complexes and the colored Tverberg problem. *J. Combin. Theory Ser. A*, 145 (2017), 400–425. **M21**

[26] Dj. Baralić, R. Živaljević. Colorful versions of the Lebesgue, KKM, and Hex theorem, *J. Combin. Theory Ser. A*, 146 (2017), 295–311. **M21**

[27] D. Jojić, S.T. Vrećica, R.T. Živaljević. Symmetric multiple chessboard complexes and a new theorem of Tverberg type. *J. Algebraic Combin.*, 46 (2017), 15–31. **M21**

[28] R. Živaljević. Topological methods in discrete geometry, Chapter 20 in Handbook of Discrete and Computational Geometry (Third Ed.), edited by Jacob E. Goodman, Joseph O'Rourke, and Csaba D. Tóth CRC Press LLC, Boca Raton, FL, 2017. **M13**

[29] D. Jojić, I. Nekrasov, G. Panina, R. Živaljević, Alexander r-tuples and Bier complexes, *Publ. Inst. Math. (Beograd) (N.S.)* 104(118) (2018), 1–22. **M24**

[30] F. D. Jevtić, M. Jelić, R.T. Živaljević. Cyclohedron and Kantorovich-Rubinstein polytopes, *Arnold Mathematical Journal*, April 2018, Vol. 4, 87–112. **M24**
`https://link.springer.com/journal/40598`

[31] M. Jelić, D. Jojić, M. Timotijević, S. T. Vrećica, R.T. Živaljević. Combinatorics of unavoidable complexes. *European Journal of Combinatorics*, Volume 83, January 2020.

[32] F. D. Jevtić, M. Timotijević, R.T. Živaljević. Polytopal Bier spheres and Kantorovich-Rubinstein polytopes of weighted cycles, *Discrete and Computational Geometry*, Online published 2019–11–19.

[33] D. Jojić, W. Marzantowicz, S.T. Vrećica, R.T. Živaljević. Topology of unavoidable complexes. *Journal of Fixed Point Theory and Applications*, accepted (February 27, 2020).

[34] D. Jojić, G. Panina, R. Živaljevic. A Tverberg type theorem for collectively unavoidable complexes. *Israel J. Math.*, accepted (November 21, 2019).

[35] F. D. Jevtić, R.T. Živaljević. Generalized Tonnetz and discrete Abel-Jacobi map. *Topological Methods in Non-linear Analysis*, accepted (May, 2020).

[36] Dj. Baralić, P-L Curien, M. Milićević, J. Obradović, Z. Petrić, M. Zekić, R.T. Živaljević. Proofs and surfaces. *Annals of Pure and Applied Logic*, Volume 171, Issue 9, October–November 2020.

[37] D. Jojić, G. Panina, R. Živaljević, Colored Tverberg theorem; extensions and new results, *Izvestiya Mathematics (Izvestiya R.A.N.)*, accepted 2021. M21

[38] D. Jojić, G. Panina, R. Živaljević, Optimal colored Tverberg theorems for prime powers, *Homology, Homotopy and Applications*, accepted 2021. M23

[39] D. Jojić, G. Panina, R. Živaljević, Splitting necklaces with constraints, SIAM J. Discrete Math. 2021. M23

4.3 International visibility (citations, ranking, etc.)

(1) Personal page at Math-Net.Ru

`http://www.mathnet.ru/php/person.phtml?option_lang=eng&personid=80903`

(2) List of publications on ZentralBlatt

`https://zbmath.org/authors/?q=au%3A%22zivaljevic%2C%20r%2A%22%20%7C%20au%3A%22zivaljevic%2C%20r%2A%20t%2A%22`

(3) List of publications on Google Scholar

`https://scholar.google.com/citations?user=8WbnmmwAAAAJ&hl=en`

	All Citations	Since 2016
Citations	1665	603
h-index	18	13
i10-index	32	22